

Supplementary File of “Many-Objective Problems Are Not Always Difficult for Pareto Dominance-Based Evolutionary Algorithms”

Hisao Ishibuchi¹ and Takashi Matsumoto² and Naoki Masuyama² and Yusuke Nojima²

1 Properties of Proposed Test Problems

We propose 18 types of test problems by combining three curvatures of Pareto fronts, two shapes of Pareto fronts, and three kinds of feasible regions. The property of each test problem is summarized in Table E1. Table E1 is the almost same as the Table 1 in the proposed paper. Table E1 is shown here for checking which type of test problems have which features.

Table E1. Formulations of 18 types of our test problem.

Type	Curvature	Pareto Front Shape	Feasible Region
01	Linear	Triangular	DTLZ
02	Linear	Triangular	WFG
03	Linear	Triangular	Minus-DTLZ
04	Linear	Inverted Triangular	DTLZ
05	Linear	Inverted Triangular	WFG
06	Linear	Inverted Triangular	Minus-DTLZ
07	Concave	Triangular	DTLZ
08	Concave	Triangular	WFG
09	Concave	Triangular	Minus-DTLZ.
10	Concave	Inverted Triangular	DTLZ
11	Concave	Inverted Triangular	WFG
12	Concave	Inverted Triangular	Minus-DTLZ
13	Convex	Triangular	DTLZ
14	Convex	Triangular	WFG
15	Convex	Triangular	Minus-DTLZ.
16	Convex	Inverted Triangular	DTLZ
17	Convex	Inverted Triangular	WFG
18	Convex	Inverted Triangular	Minus-DTLZ

2 Details in Statistical Tests

We examine whether NSGA-II is significantly better or worse than the other algorithm. We use Shapiro-Wilk’s test to check whether obtained quality indicator values follow a normal distribution or not. If the values do not follow a normal distribution, Wilcoxon’s rank sum test is used. In the other case, we use Bartlett’s test for examining equal variances between the values. If the variances are assumed equal, Student’s t-test is used. If not so, Welch’s t-test is used. It should be noted that all the tests are used at the 0.05 significance level. A series of statistical tests is summarized in Fig. E1.

In each table in the following sections, statistical test results are shown by “+” and “–”, which mean that the results of the corresponding algorithm are significantly better than and worse than those of NSGA-II, respectively. When there is no significant difference, “=” is assigned. The number of “+”, “=” and “–” is counted for each algorithm at the bottom of each table. The statistical comparison results are summarized in the last row.

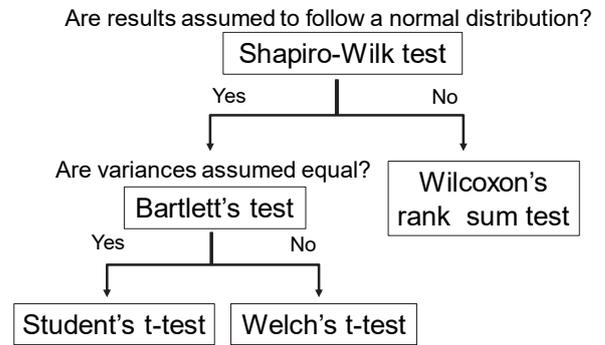


Figure E1. Statistical tests used in this paper.

3 Hypervolume ($r = 1.1$) Results.

Table E2. Average HV values ($r = 1.1$) on the 3-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.12e+00 (+)	1.12e+00 (+)	1.09e+00	1.12e+00 (+)	1.12e+00 (+)
02	1.12e+00 (+)	1.12e+00 (+)	1.10e+00	1.12e+00 (+)	1.12e+00 (+)
03	1.12e+00 (+)	1.12e+00 (+)	1.09e+00	1.12e+00 (+)	1.12e+00 (+)
04	2.75e-01 (-)	2.76e-01 (-)	2.81e-01	2.81e-01 (=)	2.69e-01 (-)
05	2.74e-01 (-)	2.76e-01 (-)	2.81e-01	2.80e-01 (-)	2.69e-01 (-)
06	2.71e-01 (-)	2.70e-01 (-)	2.75e-01	2.80e-01 (+)	2.69e-01 (-)
07	7.49e-01 (+)	7.49e-01 (+)	7.15e-01	7.49e-01 (+)	7.49e-01 (+)
08	7.46e-01 (+)	7.43e-01 (+)	7.18e-01	7.43e-01 (+)	7.48e-01 (+)
09	7.47e-01 (+)	7.28e-01 (+)	7.10e-01	7.49e-01 (+)	7.49e-01 (+)
10	5.94e-02 (-)	6.23e-02 (-)	7.62e-02	7.22e-02 (-)	5.96e-02 (-)
11	5.99e-02 (-)	6.60e-02 (-)	7.63e-02	7.23e-02 (-)	6.32e-02 (-)
12	5.89e-02 (-)	7.28e-02 (-)	7.35e-02	7.10e-02 (-)	6.55e-02 (-)
13	1.30e+00 (=)	1.30e+00 (=)	1.29e+00	1.31e+00 (+)	1.30e+00 (+)
14	1.30e+00 (-)	1.30e+00 (-)	1.30e+00	1.31e+00 (+)	1.30e+00 (-)
15	1.30e+00 (-)	1.30e+00 (-)	1.30e+00	1.31e+00 (+)	1.30e+00 (-)
16	7.02e-01 (=)	7.03e-01 (=)	7.02e-01	7.01e-01 (-)	6.99e-01 (-)
17	7.00e-01 (-)	7.03e-01 (=)	7.02e-01	6.99e-01 (-)	6.97e-01 (-)
18	7.01e-01 (+)	7.02e-01 (+)	6.94e-01	6.93e-01 (-)	6.95e-01 (=)
+/-/-	7/2/9	7/3/8	-	10/1/7	7/1/10

Table E3. Average HV values ($r = 1.1$) on the 5-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.58e+00 (+)	1.58e+00 (+)	1.33e+00	1.58e+00 (+)	1.58e+00 (+)
02	1.58e+00 (+)	1.58e+00 (+)	1.55e+00	1.58e+00 (+)	1.58e+00 (+)
03	1.58e+00 (+)	1.58e+00 (+)	1.52e+00	1.58e+00 (+)	1.58e+00 (+)
04	1.01e-02 (-)	1.80e-02 (+)	1.54e-02	1.36e-02 (-)	9.12e-03 (-)
05	8.02e-03 (-)	1.79e-02 (+)	1.63e-02	1.36e-02 (-)	9.16e-03 (-)
06	9.50e-03 (-)	1.78e-02 (+)	1.50e-02	9.38e-03 (-)	9.21e-03 (-)
07	1.31e+00 (+)	1.31e+00 (+)	1.13e+00	1.31e+00 (+)	1.31e+00 (+)
08	1.31e+00 (+)	1.31e+00 (+)	1.18e+00	1.28e+00 (+)	1.31e+00 (+)
09	1.31e+00 (+)	1.31e+00 (+)	1.14e+00	1.31e+00 (+)	1.31e+00 (+)
10	8.25e-04 (-)	2.94e-04 (-)	1.78e-03	1.03e-03 (-)	3.53e-04 (-)
11	7.31e-04 (-)	4.55e-04 (-)	1.77e-03	1.06e-03 (-)	6.56e-04 (-)
12	8.38e-04 (-)	7.17e-04 (-)	1.56e-03	9.18e-04 (-)	6.50e-04 (-)
13	1.60e+00 (+)	1.60e+00 (+)	1.45e+00	1.61e+00 (+)	1.60e+00 (+)
14	1.60e+00 (-)	1.60e+00 (-)	1.61e+00	1.61e+00 (=)	1.60e+00 (-)
15	1.60e+00 (-)	1.60e+00 (-)	1.61e+00	1.61e+00 (+)	1.61e+00 (-)
16	1.17e-01 (-)	1.59e-01 (=)	1.58e-01	1.34e-01 (-)	1.35e-01 (-)
17	1.06e-01 (-)	1.59e-01 (-)	1.78e-01	1.37e-01 (-)	1.43e-01 (-)
18	1.06e-01 (-)	1.59e-01 (-)	1.71e-01	1.26e-01 (-)	1.39e-01 (-)
+/-/-	7/0/11	10/1/7	-	8/1/9	7/0/11

Table E4. Average HV values ($r = 1.1$) on the 8-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	2.14e+00 (+)	2.14e+00 (+)	0.00e+00	2.12e+00 (+)	2.13e+00 (+)
02	2.13e+00 (+)	2.13e+00 (+)	2.09e+00	2.13e+00 (+)	2.14e+00 (+)
03	2.14e+00 (+)	2.14e+00 (+)	2.06e+00	2.14e+00 (+)	2.14e+00 (+)
04	1.72e-05 (-)	4.60e-06 (-)	2.39e-05	5.88e-05 (+)	4.42e-05 (+)
05	1.46e-05 (-)	5.39e-06 (-)	2.84e-05	5.92e-05 (+)	4.39e-05 (+)
06	1.48e-05 (-)	9.51e-06 (-)	3.18e-05	5.36e-05 (+)	4.61e-05 (+)
07	1.98e+00 (+)	1.97e+00 (+)	0.00e+00	1.93e+00 (+)	1.94e+00 (+)
08	1.97e+00 (+)	1.95e+00 (+)	1.62e+00	1.87e+00 (+)	1.97e+00 (+)
09	1.98e+00 (+)	1.97e+00 (+)	1.62e+00	1.96e+00 (+)	1.98e+00 (+)
10	6.45e-07 (-)	1.79e-07 (-)	2.60e-06	8.62e-07 (-)	4.54e-07 (-)
11	6.01e-07 (-)	1.61e-07 (-)	2.59e-06	9.38e-07 (-)	7.39e-07 (-)
12	6.01e-07 (-)	5.97e-07 (-)	2.23e-06	1.22e-06 (-)	6.96e-07 (-)
13	2.09e+00 (+)	2.08e+00 (+)	0.00e+00	2.14e+00 (+)	2.13e+00 (+)
14	2.09e+00 (-)	2.08e+00 (-)	2.14e+00	2.14e+00 (-)	2.13e+00 (-)
15	2.09e+00 (-)	2.08e+00 (-)	2.14e+00	2.14e+00 (-)	2.13e+00 (-)
16	3.32e-03 (+)	5.76e-03 (+)	1.58e-03	6.02e-03 (+)	5.21e-03 (+)
17	3.43e-03 (+)	5.91e-03 (+)	2.96e-03	6.00e-03 (+)	5.16e-03 (+)
18	3.27e-03 (-)	5.92e-03 (+)	3.99e-03	4.85e-03 (+)	5.11e-03 (+)
+/-/-	9/0/9	10/0/8	-	13/0/5	13/0/5-

Table E5. Average HV values ($r = 1.1$) on the 10-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	2.59e+00 (+)	2.59e+00 (+)	0.00e+00	2.59e+00 (+)	2.59e+00 (+)
02	2.59e+00 (+)	2.58e+00 (+)	2.56e+00	2.59e+00 (+)	2.59e+00 (+)
03	2.59e+00 (+)	2.59e+00 (+)	2.54e+00	2.59e+00 (+)	2.59e+00 (+)
04	2.22e-07 (-)	4.04e-08 (-)	3.97e-07	1.43e-06 (+)	9.06e-07 (+)
05	1.49e-07 (-)	4.75e-08 (-)	5.06e-07	1.38e-06 (+)	8.98e-07 (+)
06	1.01e-07 (-)	9.51e-08 (-)	6.64e-07	1.16e-06 (+)	7.44e-07 (+)
07	2.52e+00 (+)	2.48e+00 (+)	0.00e+00	2.38e+00 (+)	2.48e+00 (+)
08	2.51e+00 (+)	2.48e+00 (+)	2.08e+00	2.46e+00 (+)	2.51e+00 (+)
09	2.52e+00 (+)	2.51e+00 (+)	2.12e+00	2.50e+00 (+)	2.52e+00 (+)
10	6.27e-09 (-)	1.95e-09 (-)	3.90e-08	8.49e-09 (-)	5.01e-09 (-)
11	5.82e-09 (-)	1.40e-09 (-)	3.91e-08	8.87e-09 (-)	6.53e-09 (-)
12	5.76e-09 (-)	6.21e-09 (-)	3.39e-08	1.40e-08 (-)	6.55e-09 (-)
13	2.53e+00 (+)	2.51e+00 (+)	0.00e+00	2.59e+00 (+)	2.59e+00 (+)
14	2.53e+00 (-)	2.51e+00 (-)	2.59e+00	2.59e+00 (-)	2.58e+00 (-)
15	2.53e+00 (-)	2.51e+00 (-)	2.59e+00	2.59e+00 (-)	2.59e+00 (-)
16	2.25e-04 (+)	5.10e-04 (+)	1.06e-04	8.17e-04 (+)	6.64e-04 (+)
17	2.47e-04 (+)	5.36e-04 (+)	1.87e-04	8.10e-04 (+)	6.60e-04 (+)
18	2.43e-04 (-)	5.30e-04 (+)	3.55e-04	6.43e-04 (+)	6.31e-04 (+)
+/-/-	9/0/9	10/0/8	-	13/0/5	13/0/5

4 Hypervolume ($r = 1 + 1/H$) Results.

Table E6. Average HV values ($r = 1 + 1/H$) on the 3-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.04e+00 (+)	1.04e+00 (+)	1.01e+00	1.04e+00 (+)	1.04e+00 (+)
02	1.04e+00 (+)	1.04e+00 (+)	1.02e+00	1.04e+00 (+)	1.04e+00 (+)
03	1.04e+00 (+)	1.04e+00 (+)	1.01e+00	1.04e+00 (+)	1.04e+00 (+)
04	2.31e-01 (-)	2.31e-01 (-)	2.39e-01	2.37e-01 (-)	2.27e-01 (-)
05	2.31e-01 (-)	2.31e-01 (-)	2.38e-01	2.36e-01 (-)	2.27e-01 (-)
06	2.28e-01 (-)	2.25e-01 (-)	2.33e-01	2.36e-01 (+)	2.27e-01 (-)
07	6.67e-01 (+)	6.67e-01 (+)	6.33e-01	6.67e-01 (+)	6.67e-01 (+)
08	6.64e-01 (+)	6.61e-01 (+)	6.36e-01	6.61e-01 (+)	6.67e-01 (+)
09	6.65e-01 (+)	6.46e-01 (+)	6.28e-01	6.67e-01 (+)	6.67e-01 (+)
10	4.06e-02 (-)	4.31e-02 (-)	5.38e-02	5.08e-02 (-)	4.09e-02 (-)
11	4.10e-02 (-)	4.59e-02 (-)	5.39e-02	5.08e-02 (-)	4.37e-02 (-)
12	4.01e-02 (-)	5.06e-02 (-)	5.17e-02	4.97e-02 (-)	4.53e-02 (-)
13	1.22e+00 (=)	1.22e+00 (=)	1.21e+00	1.23e+00 (+)	1.22e+00 (+)
14	1.22e+00 (-)	1.22e+00 (-)	1.22e+00	1.23e+00 (+)	1.22e+00 (-)
15	1.22e+00 (-)	1.22e+00 (-)	1.22e+00	1.23e+00 (+)	1.22e+00 (-)
16	6.38e-01 (=)	6.38e-01 (=)	6.38e-01	6.37e-01 (-)	6.36e-01 (-)
17	6.36e-01 (-)	6.38e-01 (=)	6.38e-01	6.35e-01 (-)	6.34e-01 (-)
18	6.37e-01 (+)	6.37e-01 (+)	6.30e-01	6.29e-01 (=)	6.32e-01 (=)
+/=/-	7/2/9	7/3/8	-	10/1/7	7/1/10

Table E7. Average HV values ($r = 1 + 1/H$) on the 5-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	2.13e+00 (+)	2.13e+00 (+)	1.86e+00	2.13e+00 (+)	2.13e+00 (+)
02	2.13e+00 (+)	2.13e+00 (+)	2.10e+00	2.13e+00 (+)	2.13e+00 (+)
03	2.13e+00 (+)	2.13e+00 (+)	2.07e+00	2.13e+00 (+)	2.13e+00 (+)
04	3.16e-02 (-)	5.02e-02 (+)	4.70e-02	4.28e-02 (-)	2.87e-02 (-)
05	2.62e-02 (-)	5.02e-02 (+)	4.86e-02	4.28e-02 (-)	2.89e-02 (-)
06	3.01e-02 (-)	5.03e-02 (+)	4.49e-02	3.24e-02 (-)	2.90e-02 (-)
07	1.86e+00 (+)	1.86e+00 (+)	1.67e+00	1.86e+00 (+)	1.86e+00 (+)
08	1.86e+00 (+)	1.86e+00 (+)	1.73e+00	1.83e+00 (+)	1.86e+00 (+)
09	1.86e+00 (+)	1.86e+00 (+)	1.70e+00	1.86e+00 (+)	1.86e+00 (+)
10	5.44e-03 (-)	2.00e-03 (-)	9.62e-03	6.32e-03 (-)	2.59e-03 (-)
11	4.76e-03 (-)	2.92e-03 (-)	9.54e-03	6.45e-03 (-)	4.33e-03 (-)
12	5.50e-03 (-)	4.85e-03 (-)	8.68e-03	5.78e-03 (-)	4.34e-03 (-)
13	2.15e+00 (+)	2.15e+00 (+)	1.99e+00	2.16e+00 (+)	2.15e+00 (+)
14	2.15e+00 (-)	2.14e+00 (-)	2.16e+00	2.16e+00 (=)	2.16e+00 (-)
15	2.15e+00 (-)	2.15e+00 (-)	2.16e+00	2.16e+00 (+)	2.16e+00 (-)
16	2.40e-01 (-)	2.96e-01 (+)	2.92e-01	2.67e-01 (-)	2.59e-01 (-)
17	2.18e-01 (-)	2.97e-01 (-)	3.21e-01	2.71e-01 (-)	2.70e-01 (-)
18	2.24e-01 (-)	2.97e-01 (-)	3.08e-01	2.50e-01 (-)	2.64e-01 (-)
+/=/-	7/0/11	11/0/7	-	8/1/9	7/0/11

Table E8. Average HV values ($r = 1 + 1/H$) on the 8-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	9.98e+00 (+)	9.98e+00 (+)	0.00e+00	9.95e+00 (+)	9.97e+00 (+)
02	9.97e+00 (+)	9.93e+00 (=)	9.94e+00	9.96e+00 (+)	9.97e+00 (+)
03	9.98e+00 (+)	9.98e+00 (+)	9.90e+00	9.98e+00 (+)	9.98e+00 (+)
04	6.59e-03 (-)	4.59e-03 (-)	1.80e-02	1.17e-02 (-)	9.09e-03 (-)
05	6.07e-03 (-)	5.05e-03 (-)	1.93e-02	1.20e-02 (-)	9.32e-03 (-)
06	6.12e-03 (-)	6.60e-03 (-)	1.91e-02	1.18e-02 (-)	9.99e-03 (-)
07	9.83e+00 (+)	9.77e+00 (+)	0.00e+00	9.75e+00 (+)	9.78e+00 (+)
08	9.80e+00 (+)	9.69e+00 (+)	9.46e+00	9.60e+00 (+)	9.80e+00 (+)
09	9.82e+00 (+)	9.81e+00 (+)	9.46e+00	9.79e+00 (+)	9.83e+00 (+)
10	1.85e-03 (-)	8.01e-04 (-)	6.22e-03	1.98e-03 (-)	1.39e-03 (-)
11	1.79e-03 (-)	7.43e-04 (-)	6.19e-03	2.15e-03 (-)	2.07e-03 (-)
12	1.79e-03 (-)	2.18e-03 (-)	5.67e-03	3.17e-03 (-)	1.93e-03 (-)
13	9.78e+00 (+)	9.75e+00 (+)	0.00e+00	9.99e+00 (+)	9.97e+00 (+)
14	9.78e+00 (-)	9.73e+00 (-)	9.99e+00	9.97e+00 (-)	9.96e+00 (-)
15	9.80e+00 (-)	9.74e+00 (-)	9.99e+00	9.99e+00 (-)	9.97e+00 (-)
16	1.16e-01 (-)	1.30e-01 (+)	1.20e-01	1.86e-01 (+)	1.46e-01 (+)
17	1.19e-01 (-)	1.31e-01 (-)	1.67e-01	1.85e-01 (+)	1.44e-01 (-)
18	1.14e-01 (-)	1.31e-01 (-)	1.87e-01	1.66e-01 (-)	1.45e-01 (-)
+/=/-	7/0/11	7/1/10	-	9/0/9	8/0/10

Table E9. Average HV values ($r = 1 + 1/H$) on the 10-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.78e+01 (+)	1.78e+01 (+)	0.00e+00	1.78e+01 (+)	1.78e+01 (+)
02	1.77e+01 (+)	1.77e+01 (=)	1.77e+01	1.77e+01 (+)	1.77e+01 (+)
03	1.78e+01 (+)	1.78e+01 (+)	1.77e+01	1.78e+01 (+)	1.78e+01 (+)
04	7.27e-04 (-)	4.78e-04 (-)	3.07e-03	1.78e-03 (-)	1.28e-03 (-)
05	6.58e-04 (-)	5.16e-04 (-)	3.41e-03	1.81e-03 (-)	1.21e-03 (-)
06	5.36e-04 (-)	7.33e-04 (-)	3.61e-03	1.67e-03 (-)	1.01e-03 (-)
07	1.77e+01 (+)	1.75e+01 (+)	0.00e+00	1.75e+01 (+)	1.76e+01 (+)
08	1.77e+01 (+)	1.75e+01 (+)	1.72e+01	1.76e+01 (+)	1.77e+01 (+)
09	1.77e+01 (+)	1.77e+01 (+)	1.73e+01	1.76e+01 (+)	1.77e+01 (+)
10	2.22e-04 (-)	9.42e-05 (-)	9.57e-04	2.12e-04 (-)	1.65e-04 (-)
11	2.14e-04 (-)	7.65e-05 (-)	9.55e-04	2.22e-04 (-)	2.09e-04 (-)
12	2.11e-04 (-)	2.69e-04 (-)	8.71e-04	3.97e-04 (-)	2.23e-04 (-)
13	1.74e+01 (+)	1.73e+01 (+)	0.00e+00	1.78e+01 (+)	1.77e+01 (+)
14	1.74e+01 (-)	1.73e+01 (-)	1.78e+01	1.77e+01 (-)	1.77e+01 (-)
15	1.74e+01 (-)	1.73e+01 (-)	1.78e+01	1.78e+01 (-)	1.77e+01 (-)
16	2.38e-02 (-)	3.53e-02 (=)	3.62e-02	6.79e-02 (+)	5.12e-02 (+)
17	2.51e-02 (-)	3.54e-02 (-)	5.14e-02	6.99e-02 (+)	5.05e-02 (=)
18	2.45e-02 (-)	3.56e-02 (-)	6.64e-02	6.08e-02 (-)	4.87e-02 (-)
+/=/-	7/0/11	6/2/10	-	9/0/9	8/1/9

5 IGD Results.

Table E10. Average IGD values on the 3-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	3.80e-02 (+)	3.80e-02 (+)	5.69e-02	3.80e-02 (+)	3.80e-02 (+)
02	3.80e-02 (+)	3.81e-02 (+)	5.17e-02	3.80e-02 (+)	3.80e-02 (+)
03	3.80e-02 (+)	3.83e-02 (+)	5.35e-02	3.79e-02 (+)	3.79e-02 (+)
04	5.98e-02 (-)	6.17e-02 (-)	5.29e-02	5.54e-02 (-)	6.36e-02 (-)
05	5.97e-02 (-)	6.16e-02 (-)	5.29e-02	5.57e-02 (-)	6.34e-02 (-)
06	6.22e-02 (-)	6.50e-02 (-)	5.34e-02	5.58e-02 (-)	6.37e-02 (-)
07	5.03e-02 (+)	5.03e-02 (+)	6.74e-02	5.03e-02 (+)	5.03e-02 (+)
08	5.03e-02 (+)	5.16e-02 (+)	6.62e-02	5.88e-02 (+)	5.03e-02 (+)
09	5.13e-02 (+)	5.71e-02 (+)	6.67e-02	5.03e-02 (+)	5.03e-02 (+)
10	7.47e-02 (-)	1.18e-01 (-)	4.51e-02	5.05e-02 (-)	1.11e-01 (-)
11	7.35e-02 (-)	1.01e-01 (-)	4.54e-02	4.90e-02 (-)	7.67e-02 (-)
12	7.42e-02 (-)	6.52e-02 (-)	3.98e-02	4.54e-02 (-)	6.17e-02 (-)
13	5.85e-02 (=)	7.41e-02 (-)	6.17e-02	4.23e-02 (+)	6.03e-02 (=)
14	5.76e-02 (-)	7.40e-02 (-)	4.48e-02	4.20e-02 (+)	6.02e-02 (-)
15	5.84e-02 (-)	7.43e-02 (-)	4.16e-02	4.20e-02 (-)	6.03e-02 (-)
16	6.90e-02 (-)	6.76e-02 (-)	6.59e-02	6.93e-02 (-)	8.02e-02 (-)
17	6.91e-02 (-)	6.77e-02 (-)	6.57e-02	6.93e-02 (-)	8.18e-02 (-)
18	6.88e-02 (-)	6.76e-02 (-)	6.56e-02	6.81e-02 (-)	8.06e-02 (-)
+/-/-	6/1/11	6/0/12	-	8/0/10	6/1/11

Table E11. Average IGD values on the 5-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.05e-01 (+)	1.05e-01 (+)	2.98e-01	1.05e-01 (+)	1.05e-01 (+)
02	1.05e-01 (+)	1.05e-01 (+)	1.33e-01	1.05e-01 (+)	1.05e-01 (+)
03	1.05e-01 (+)	1.05e-01 (+)	1.47e-01	1.05e-01 (+)	1.05e-01 (+)
04	2.00e-01 (-)	1.26e-01 (+)	1.38e-01	1.67e-01 (-)	2.11e-01 (-)
05	2.57e-01 (-)	1.27e-01 (+)	1.32e-01	1.66e-01 (-)	2.10e-01 (-)
06	2.07e-01 (-)	1.27e-01 (+)	1.30e-01	1.81e-01 (-)	2.10e-01 (-)
07	1.65e-01 (+)	1.65e-01 (+)	2.29e-01	1.65e-01 (+)	1.65e-01 (+)
08	1.65e-01 (+)	1.65e-01 (+)	1.85e-01	1.91e-01 (-)	1.65e-01 (+)
09	1.66e-01 (+)	1.66e-01 (+)	1.94e-01	1.65e-01 (+)	1.65e-01 (+)
10	2.46e-01 (-)	4.04e-01 (-)	7.96e-02	1.74e-01 (-)	6.20e-01 (-)
11	2.61e-01 (-)	3.16e-01 (-)	7.93e-02	1.72e-01 (-)	2.62e-01 (-)
12	2.38e-01 (-)	2.70e-01 (-)	6.98e-02	1.44e-01 (-)	2.63e-01 (-)
13	1.03e-01 (+)	1.06e-01 (+)	2.54e-01	6.08e-02 (+)	9.69e-02 (+)
14	1.01e-01 (-)	1.05e-01 (-)	7.86e-02	6.01e-02 (+)	9.55e-02 (-)
15	1.02e-01 (-)	1.05e-01 (-)	7.77e-02	6.01e-02 (+)	9.59e-02 (-)
16	2.30e-01 (-)	2.32e-01 (-)	1.95e-01	2.03e-01 (-)	2.50e-01 (-)
17	2.64e-01 (-)	2.32e-01 (-)	1.83e-01	2.01e-01 (-)	2.46e-01 (-)
18	2.39e-01 (-)	2.32e-01 (-)	1.85e-01	1.98e-01 (-)	2.43e-01 (-)
+/-/-	7/0/11	10/0/8	-	8/0/10	7/0/11

Table E12. Average IGD values on the 8-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.94e-01 (+)	1.94e-01 (+)	2.37e+02	2.49e-01 (+)	2.09e-01 (+)
02	1.95e-01 (+)	1.94e-01 (+)	2.88e-01	2.04e-01 (+)	1.94e-01 (+)
03	1.98e-01 (+)	1.94e-01 (+)	2.99e-01	1.95e-01 (+)	1.94e-01 (+)
04	3.89e-01 (-)	5.00e-01 (-)	2.64e-01	2.81e-01 (-)	3.12e-01 (-)
05	4.05e-01 (-)	4.87e-01 (-)	2.53e-01	2.79e-01 (-)	3.09e-01 (-)
06	3.97e-01 (-)	4.35e-01 (-)	2.33e-01	2.73e-01 (-)	3.04e-01 (-)
07	3.15e-01 (+)	3.18e-01 (+)	3.28e+02	3.44e-01 (+)	3.33e-01 (+)
08	3.16e-01 (+)	3.30e-01 (+)	4.33e-01	4.09e-01 (=)	3.15e-01 (+)
09	3.33e-01 (+)	3.26e-01 (+)	4.21e-01	3.59e-01 (+)	3.41e-01 (+)
10	3.32e-01 (-)	6.77e-01 (-)	1.21e-01	1.16e-01 (+)	3.32e-01 (-)
11	3.63e-01 (-)	7.33e-01 (-)	1.19e-01	1.16e-01 (=)	2.77e-01 (-)
12	3.68e-01 (-)	3.94e-01 (-)	1.07e-01	1.83e-01 (-)	3.11e-01 (-)
13	1.19e-01 (+)	1.45e-01 (+)	5.57e+01	1.01e-01 (+)	1.37e-01 (+)
14	1.17e-01 (+)	1.45e-01 (-)	1.20e-01	1.00e-01 (+)	1.24e-01 (-)
15	1.19e-01 (+)	1.45e-01 (-)	1.26e-01	9.13e-02 (+)	1.31e-01 (-)
16	6.36e-01 (-)	6.08e-01 (-)	4.16e-01	5.34e-01 (-)	6.19e-01 (-)
17	6.36e-01 (-)	6.06e-01 (-)	3.79e-01	5.33e-01 (-)	6.21e-01 (-)
18	6.36e-01 (-)	6.07e-01 (-)	3.68e-01	4.96e-01 (-)	6.09e-01 (-)
+/-/-	9/0/9	7/0/11	-	9/2/7	7/0/11

Table E13. Average IGD values on the 10-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	2.18e-01 (+)	2.19e-01 (+)	3.64e+02	2.60e-01 (+)	2.32e-01 (+)
02	2.19e-01 (+)	2.19e-01 (+)	3.27e-01	2.22e-01 (+)	2.17e-01 (+)
03	2.23e-01 (+)	2.19e-01 (+)	3.29e-01	2.30e-01 (+)	2.17e-01 (+)
04	4.12e-01 (-)	5.41e-01 (-)	2.85e-01	2.77e-01 (+)	3.19e-01 (-)
05	4.41e-01 (-)	5.29e-01 (-)	2.71e-01	2.77e-01 (-)	3.19e-01 (-)
06	4.65e-01 (-)	4.71e-01 (-)	2.43e-01	2.80e-01 (-)	3.32e-01 (-)
07	4.21e-01 (+)	4.30e-01 (+)	5.99e+02	4.90e-01 (+)	4.35e-01 (+)
08	4.21e-01 (+)	4.31e-01 (+)	4.89e-01	4.65e-01 (=)	4.20e-01 (+)
09	4.31e-01 (+)	4.37e-01 (+)	4.60e-01	4.54e-01 (+)	4.38e-01 (+)
10	3.62e-01 (-)	6.65e-01 (-)	1.20e-01	1.03e-01 (+)	3.11e-01 (-)
11	3.87e-01 (-)	7.87e-01 (-)	1.19e-01	1.01e-01 (+)	2.96e-01 (-)
12	3.95e-01 (-)	5.60e-01 (-)	1.09e-01	1.77e-01 (-)	3.76e-01 (-)
13	1.06e-01 (+)	1.26e-01 (+)	1.26e+02	1.01e-01 (+)	1.16e-01 (+)
14	1.05e-01 (+)	1.26e-01 (-)	1.22e-01	7.82e-02 (+)	1.07e-01 (+)
15	1.06e-01 (+)	1.26e-01 (=)	1.28e-01	7.49e-02 (+)	1.09e-01 (+)
16	7.39e-01 (-)	6.88e-01 (-)	4.56e-01	5.97e-01 (-)	6.66e-01 (-)
17	7.37e-01 (-)	6.86e-01 (-)	4.18e-01	5.87e-01 (-)	6.68e-01 (-)
18	7.35e-01 (-)	6.83e-01 (-)	3.98e-01	5.58e-01 (-)	6.66e-01 (-)
+/-/-	9/0/9	7/1/10	-	11/1/6	9/0/9

6 IGD⁺ Results.

Table E14. Average IGD⁺ values on the 3-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	2.69e-02 (+)	2.70e-02 (+)	3.93e-02	2.70e-02 (+)	2.69e-02 (+)
02	2.77e-02 (+)	2.79e-02 (+)	3.55e-02	2.72e-02 (+)	2.71e-02 (+)
03	2.76e-02 (+)	2.83e-02 (+)	4.05e-02	2.68e-02 (+)	2.68e-02 (+)
04	4.12e-02 (-)	4.32e-02 (-)	3.69e-02	3.84e-02 (-)	4.40e-02 (-)
05	4.15e-02 (-)	4.32e-02 (-)	3.70e-02	3.87e-02 (-)	4.39e-02 (-)
06	4.32e-02 (-)	4.78e-02 (-)	4.06e-02	3.91e-02 (+)	4.40e-02 (-)
07	2.10e-02 (+)	2.12e-02 (+)	3.12e-02	2.11e-02 (+)	2.11e-02 (+)
08	2.21e-02 (+)	2.33e-02 (+)	3.04e-02	2.52e-02 (+)	2.11e-02 (+)
09	2.18e-02 (+)	2.80e-02 (+)	3.34e-02	2.09e-02 (+)	2.09e-02 (+)
10	3.44e-02 (-)	6.38e-02 (-)	1.64e-02	1.93e-02 (-)	4.25e-02 (-)
11	3.37e-02 (-)	4.91e-02 (-)	1.63e-02	1.90e-02 (-)	3.26e-02 (-)
12	3.51e-02 (-)	2.06e-02 (-)	1.79e-02	2.03e-02 (-)	2.74e-02 (-)
13	1.18e-02 (+)	1.41e-02 (+)	3.32e-02	2.01e-02 (+)	1.42e-02 (+)
14	1.22e-02 (+)	1.46e-02 (+)	2.98e-02	1.99e-02 (+)	1.45e-02 (+)
15	1.19e-02 (+)	1.41e-02 (+)	2.76e-02	1.96e-02 (+)	1.42e-02 (+)
16	3.28e-02 (-)	3.31e-02 (-)	3.14e-02	3.45e-02 (-)	3.38e-02 (-)
17	3.34e-02 (-)	3.33e-02 (-)	3.15e-02	3.51e-02 (-)	3.44e-02 (-)
18	3.32e-02 (+)	3.34e-02 (+)	3.45e-02	3.73e-02 (-)	3.57e-02 (-)
+/-/-	10/0/8	10/0/8	-	10/0/8	9/0/9

Table E14. Average IGD⁺ values on the 5-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	7.51e-02 (+)	7.52e-02 (+)	2.06e-01	7.52e-02 (+)	7.51e-02 (+)
02	7.52e-02 (+)	7.54e-02 (+)	1.01e-01	7.52e-02 (+)	7.52e-02 (+)
03	7.51e-02 (+)	7.51e-02 (+)	1.22e-01	7.51e-02 (+)	7.51e-02 (+)
04	1.32e-01 (-)	8.47e-02 (+)	9.82e-02	1.09e-01 (-)	1.40e-01 (-)
05	1.81e-01 (-)	8.53e-02 (+)	9.38e-02	1.09e-01 (-)	1.39e-01 (-)
06	1.38e-01 (-)	8.52e-02 (+)	1.01e-01	1.40e-01 (-)	1.40e-01 (-)
07	6.21e-02 (+)	6.24e-02 (+)	1.17e-01	6.21e-02 (+)	6.21e-02 (+)
08	6.22e-02 (+)	6.25e-02 (+)	1.02e-01	7.37e-02 (+)	6.21e-02 (+)
09	6.22e-02 (+)	6.23e-02 (+)	1.11e-01	6.20e-02 (+)	6.20e-02 (+)
10	6.14e-02 (-)	1.92e-01 (-)	2.66e-02	5.26e-02 (-)	1.54e-01 (-)
11	9.02e-02 (-)	1.65e-01 (-)	2.70e-02	5.04e-02 (-)	1.01e-01 (-)
12	5.99e-02 (-)	7.55e-02 (-)	2.96e-02	5.62e-02 (-)	9.40e-02 (-)
13	1.53e-02 (+)	1.59e-02 (+)	1.87e-01	3.64e-02 (+)	1.60e-02 (+)
14	1.55e-02 (+)	1.61e-02 (+)	6.02e-02	3.58e-02 (+)	1.63e-02 (+)
15	1.53e-02 (+)	1.58e-02 (+)	6.22e-02	3.55e-02 (+)	1.62e-02 (+)
16	1.39e-01 (-)	1.28e-01 (-)	1.15e-01	1.26e-01 (-)	1.34e-01 (-)
17	1.67e-01 (-)	1.28e-01 (-)	1.00e-01	1.25e-01 (-)	1.29e-01 (-)
18	1.48e-01 (-)	1.28e-01 (-)	1.08e-01	1.35e-01 (-)	1.31e-01 (-)
+/-/-	9/0/9	12/0/6	-	9/0/9	9/0/9

Table E15. Average IGD⁺ values on the 8-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.27e-01 (+)	1.28e-01 (+)	2.37e+02	1.63e-01 (+)	1.38e-01 (+)
02	1.30e-01 (+)	1.30e-01 (+)	2.29e-01	1.32e-01 (+)	1.28e-01 (+)
03	1.38e-01 (+)	1.30e-01 (+)	2.48e-01	1.29e-01 (+)	1.28e-01 (+)
04	2.91e-01 (-)	3.71e-01 (-)	1.89e-01	2.08e-01 (-)	2.36e-01 (-)
05	3.04e-01 (-)	3.59e-01 (-)	1.82e-01	2.07e-01 (-)	2.33e-01 (-)
06	2.99e-01 (-)	3.20e-01 (-)	1.79e-01	2.08e-01 (-)	2.28e-01 (-)
07	1.22e-01 (+)	1.26e-01 (+)	3.28e+02	1.38e-01 (+)	1.30e-01 (+)
08	1.23e-01 (+)	1.34e-01 (+)	2.12e-01	1.79e-01 (+)	1.22e-01 (+)
09	1.24e-01 (+)	1.25e-01 (+)	2.09e-01	1.30e-01 (+)	1.20e-01 (+)
10	1.37e-01 (-)	1.96e-01 (-)	4.45e-02	9.89e-02 (-)	1.57e-01 (-)
11	1.41e-01 (-)	1.98e-01 (-)	4.45e-02	9.54e-02 (-)	1.33e-01 (-)
12	1.43e-01 (-)	1.44e-01 (-)	4.71e-02	9.75e-02 (-)	1.11e-01 (-)
13	1.36e-02 (+)	1.46e-02 (+)	5.57e+01	2.69e-02 (+)	2.50e-02 (+)
14	1.40e-02 (+)	1.56e-02 (+)	9.54e-02	3.14e-02 (+)	1.54e-02 (+)
15	1.34e-02 (+)	1.46e-02 (+)	1.08e-01	1.79e-02 (+)	1.41e-02 (+)
16	3.42e-01 (-)	3.34e-01 (-)	2.73e-01	2.67e-01 (+)	3.15e-01 (-)
17	3.38e-01 (-)	3.36e-01 (-)	2.34e-01	2.69e-01 (-)	3.17e-01 (-)
18	3.44e-01 (-)	3.35e-01 (-)	2.23e-01	2.81e-01 (-)	3.16e-01 (-)
+/-/-	9/0/9	9/0/9	-	9/1/8	9/0/9

Table E16. Average IGD⁺ values on the 10-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	1.38e-01 (+)	1.39e-01 (+)	3.64e+02	1.66e-01 (+)	1.47e-01 (+)
02	1.40e-01 (+)	1.40e-01 (+)	2.69e-01	1.40e-01 (+)	1.37e-01 (+)
03	1.48e-01 (+)	1.41e-01 (+)	2.77e-01	1.54e-01 (+)	1.38e-01 (+)
04	3.08e-01 (-)	3.87e-01 (-)	1.99e-01	2.12e-01 (-)	2.43e-01 (-)
05	3.26e-01 (-)	3.79e-01 (-)	1.90e-01	2.11e-01 (-)	2.44e-01 (-)
06	3.42e-01 (-)	3.45e-01 (-)	1.82e-01	2.17e-01 (-)	2.57e-01 (-)
07	1.75e-01 (+)	1.84e-01 (+)	5.99e+02	2.13e-01 (+)	1.82e-01 (+)
08	1.75e-01 (+)	1.83e-01 (+)	2.48e-01	2.01e-01 (+)	1.75e-01 (+)
09	1.73e-01 (+)	1.80e-01 (+)	2.39e-01	1.81e-01 (+)	1.70e-01 (+)
10	1.25e-01 (-)	1.63e-01 (-)	4.16e-02	9.11e-02 (-)	1.35e-01 (-)
11	1.27e-01 (-)	1.67e-01 (-)	4.17e-02	8.80e-02 (-)	1.27e-01 (-)
12	1.29e-01 (-)	1.25e-01 (-)	4.29e-02	9.24e-02 (-)	1.10e-01 (-)
13	1.14e-02 (+)	1.15e-02 (+)	1.26e+02	3.36e-02 (+)	2.39e-02 (+)
14	1.17e-02 (+)	1.19e-02 (+)	1.03e-01	1.89e-02 (+)	1.49e-02 (+)
15	1.15e-02 (+)	1.15e-02 (+)	1.12e-01	1.54e-02 (+)	1.41e-02 (+)
16	4.20e-01 (-)	3.74e-01 (-)	3.02e-01	2.89e-01 (+)	3.32e-01 (-)
17	4.12e-01 (-)	3.73e-01 (-)	2.65e-01	2.85e-01 (-)	3.35e-01 (-)
18	4.14e-01 (-)	3.73e-01 (-)	2.44e-01	3.00e-01 (-)	3.39e-01 (-)
+/-/-	9/0/9	9/0/9	-	10/0/8	9/0/9

7 AVERAGE DISTANCE FUNCTION Results.

Table E17. Average distance function values on the 3-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	0.000 (+)	0.000 (+)	0.358	0.000 (+)	0.000 (+)
02	0.001 (-)	0.001 (-)	0.000	0.000 (-)	0.000 (=)
03	0.970 (+)	1.869 (+)	8.630	0.001 (+)	0.000 (+)
04	0.136 (-)	0.000 (+)	0.001	0.252 (=)	0.125 (-)
05	0.000 (+)	0.000 (+)	0.001	0.000 (+)	0.000 (+)
06	4.721 (+)	5.084 (+)	16.953	5.443 (+)	1.142 (+)
07	0.000 (+)	0.000 (+)	0.150	0.000 (+)	0.000 (+)
08	0.001 (-)	0.004 (-)	0.000	0.000 (=)	0.000 (=)
09	3.918 (+)	42.689 (-)	18.334	0.014 (+)	0.012 (+)
10	0.520 (=)	0.070 (=)	0.000	0.177 (-)	0.100 (=)
11	0.000 (=)	0.018 (-)	0.000	0.000 (-)	0.000 (=)
12	18.884 (+)	16.404 (+)	35.116	36.037 (=)	24.649 (+)
13	0.000 (+)	0.000 (+)	0.409	0.000 (+)	0.000 (+)
14	0.000 (-)	0.000 (-)	0.000	0.000 (-)	0.000 (-)
15	0.096 (+)	0.089 (+)	1.131	0.001 (+)	0.000 (+)
16	0.003 (-)	0.000 (+)	0.002	0.042 (=)	0.002 (=)
17	0.001 (=)	0.000 (+)	0.000	0.001 (-)	0.001 (-)
18	1.437 (+)	0.367 (+)	7.825	8.851 (-)	5.949 (+)
+/=/-	10/3/5	12/1/5	-	8/4/6	10/5/3

Table E18. Average distance function values on the 5-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	0.000 (+)	0.000 (+)	11.236	0.000 (+)	0.000 (+)
02	0.000 (+)	0.000 (+)	0.012	0.000 (+)	0.000 (+)
03	0.006 (+)	0.013 (+)	37.516	0.001 (+)	0.001 (+)
04	0.059 (-)	0.000 (+)	0.013	0.087 (-)	0.000 (+)
05	0.023 (-)	0.000 (+)	0.006	0.000 (+)	0.000 (+)
06	0.637 (+)	0.001 (+)	58.22	75.146 (-)	0.224 (+)
07	0.000 (+)	0.000 (+)	4.292	0.000 (+)	0.000 (+)
08	0.000 (+)	0.000 (+)	0.010	0.000 (+)	0.000 (+)
09	1.109 (+)	2.131 (+)	73.423	0.038 (+)	0.020 (+)
10	0.485 (=)	0.015 (-)	0.001	0.052 (-)	0.095 (-)
11	0.002 (-)	0.009 (-)	0.001	0.000 (+)	0.000 (+)
12	2.201 (+)	0.013 (+)	94.407	79.482 (+)	5.857 (+)
13	0.000 (+)	0.001 (+)	15.951	0.004 (+)	0.000 (+)
14	0.000 (-)	0.000 (-)	0.000	0.000 (-)	0.000 (-)
15	0.002 (+)	0.002 (+)	2.746	0.004 (+)	0.002 (+)
16	0.022 (+)	0.000 (+)	0.049	0.029 (+)	0.001 (+)
17	0.000 (+)	0.000 (+)	0.010	0.001 (+)	0.001 (+)
18	5.890 (+)	0.046 (+)	38.982	42.068 (-)	8.215 (+)
+/=/-	13/1/4	15/0/3	-	13/0/5	16/0/2

Table E19. Average distance function values on the 8-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	0.001 (+)	0.003 (+)	915.158	3.293 (+)	0.125 (+)
02	0.001 (+)	0.001 (+)	0.050	0.000 (+)	0.000 (+)
03	3.120 (+)	0.774 (+)	76.267	0.447 (+)	0.116 (+)
04	0.032 (+)	0.001 (+)	0.034	0.001 (+)	0.001 (+)
05	0.000 (+)	0.001 (+)	0.018	0.001 (+)	0.001 (+)
06	3.852 (+)	0.000 (+)	75.133	54.587 (+)	10.531 (+)
07	0.001 (+)	0.001 (+)	957.925	0.832 (+)	0.253 (+)
08	0.002 (+)	0.002 (+)	0.083	0.001 (+)	0.001 (+)
09	33.898 (+)	31.599 (+)	157.442	16.076 (+)	16.034 (+)
10	0.136 (-)	0.001 (+)	0.002	0.002 (+)	0.131 (-)
11	0.000 (+)	0.000 (+)	0.002	0.002 (+)	0.000 (+)
12	2.624 (+)	0.176 (+)	123.745	58.506 (+)	1.589 (+)
13	0.001 (+)	0.003 (+)	734.321	5.228 (+)	0.854 (+)
14	0.000 (+)	0.000 (+)	0.001	0.000 (+)	0.000 (+)
15	0.019 (+)	0.008 (+)	7.010	0.132 (+)	0.014 (+)
16	0.000 (+)	0.001 (+)	0.135	0.007 (+)	0.002 (+)
17	0.000 (+)	0.001 (+)	0.038	0.004 (+)	0.002 (+)
18	4.405 (+)	0.047 (+)	50.739	80.641 (-)	10.053 (+)
+/=/-	17/0/1	18/0/0	-	17/0/1	17/0/1

Table E20. Average distance function values on the 10-objective test problems. The best and worst results are shown by red and blue fonts, respectively.

Type	MOEA/DD	MOEA/D	NSGA-II	NSGA-III	θ -DEA
01	0.001 (+)	0.003 (+)	1007.490	3.826 (+)	0.169 (+)
02	0.000 (+)	0.001 (+)	0.061	0.000 (+)	0.000 (+)
03	1.814 (+)	0.413 (+)	81.961	2.152 (+)	0.198 (+)
04	0.000 (+)	0.001 (+)	0.046	0.000 (+)	0.000 (+)
05	0.000 (+)	0.000 (+)	0.022	0.000 (+)	0.000 (+)
06	0.206 (+)	0.000 (+)	69.799	53.317 (+)	2.410 (+)
07	0.000 (+)	0.001 (+)	1037.140	3.000 (+)	0.460 (+)
08	0.001 (+)	0.004 (+)	0.118	0.000 (+)	0.000 (+)
09	23.429 (+)	24.241 (+)	181.540	11.918 (+)	11.503 (+)
10	0.089 (-)	0.001 (+)	0.003	0.001 (+)	0.042 (-)
11	0.000 (+)	0.000 (+)	0.002	0.001 (+)	0.000 (+)
12	4.762 (+)	0.000 (+)	122.980	62.331 (+)	0.977 (+)
13	0.001 (+)	0.002 (+)	915.301	15.939 (+)	1.935 (+)
14	0.000 (+)	0.000 (+)	0.002	0.000 (+)	0.000 (+)
15	0.005 (+)	0.007 (+)	6.350	0.110 (+)	0.016 (+)
16	0.000 (+)	0.000 (+)	0.146	0.002 (+)	0.001 (+)
17	0.000 (+)	0.000 (+)	0.043	0.001 (+)	0.001 (+)
18	0.068 (+)	0.045 (+)	41.477	64.802 (-)	7.984 (+)
+/=/-	17/0/1	18/0/0/	-	17/0/1	17/0/1

8 Obtained Population

In this section, we show the populations which obtained the median Hypervolume value of all values with reference point $r = 1 + 1/H$.

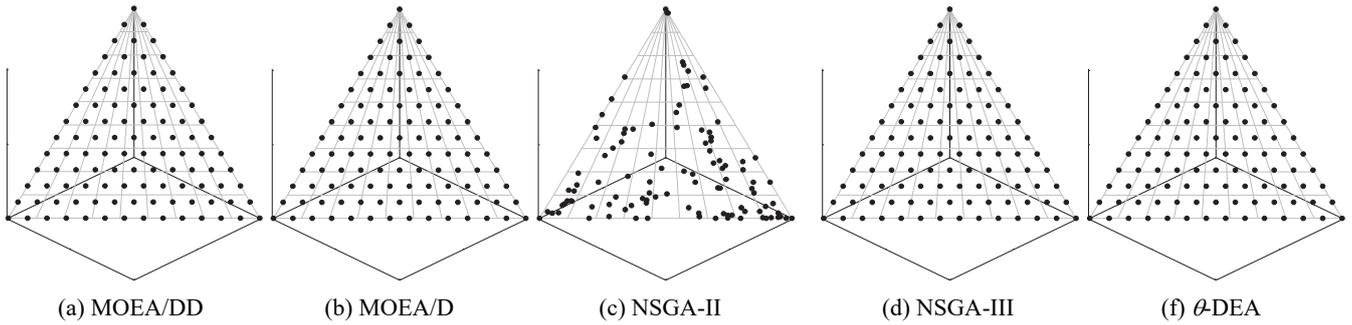


Figure E2. The obtained populations on the three-objective type 01 test problem. In type 01 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, DTLZ, respectively.

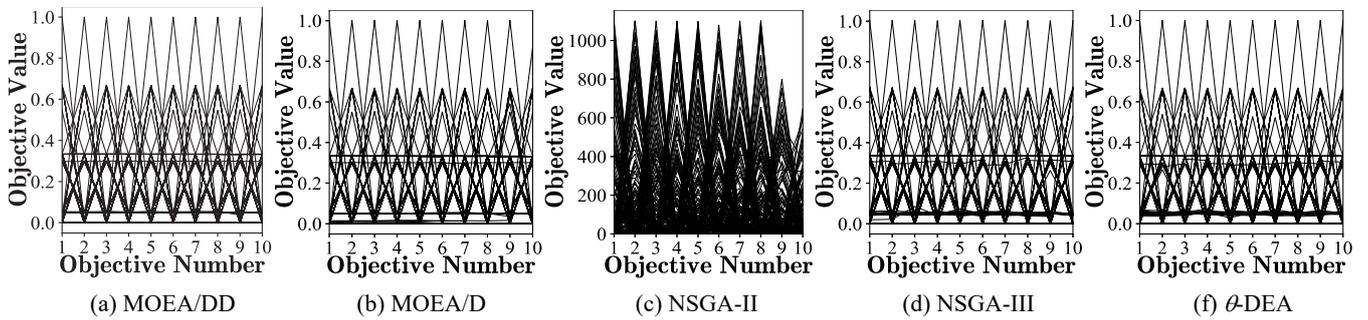


Figure E3. The obtained populations on the ten-objective type 01 test problem. In type 01 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, DTLZ, respectively.

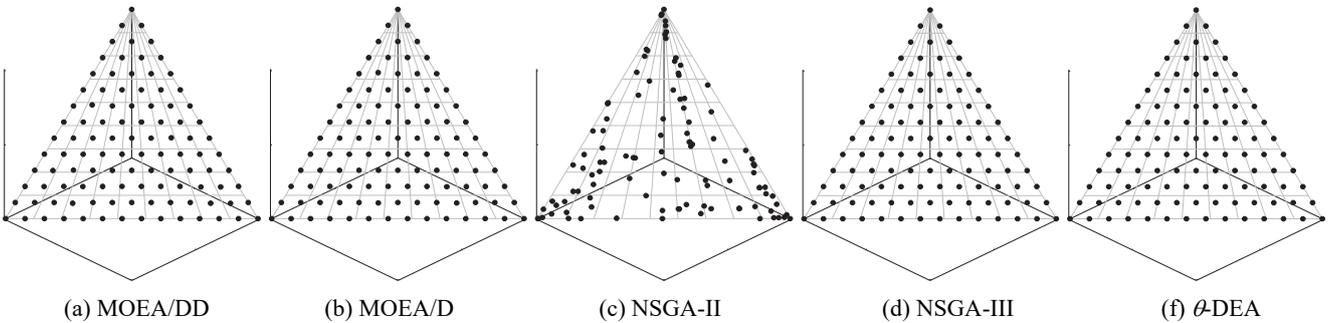


Figure E4. The obtained populations on the three-objective type 02 test problem. In type 02 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, WFG, respectively.

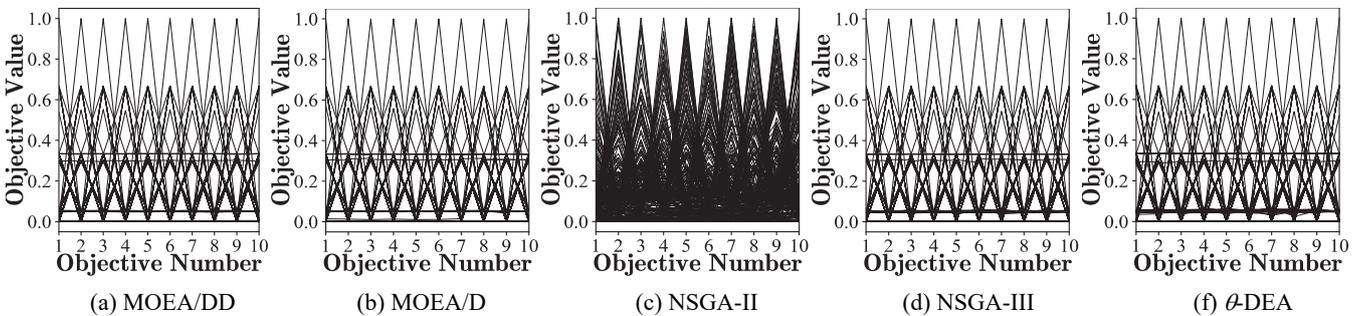


Figure E5. The obtained populations on the ten-objective type 02 test problem. In type 02 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, WFG, respectively.

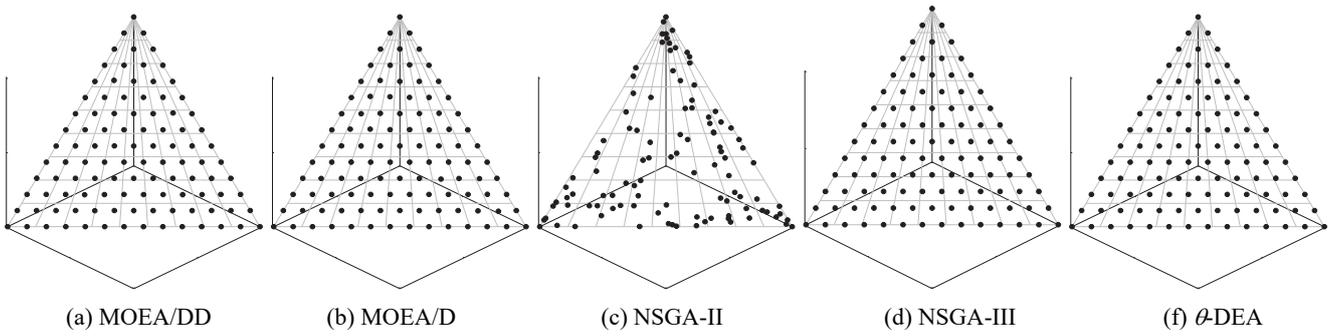


Figure E6. The obtained populations on the three-objective type 03 test problem. In type 03 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, Minus-DTLZ, respectively.

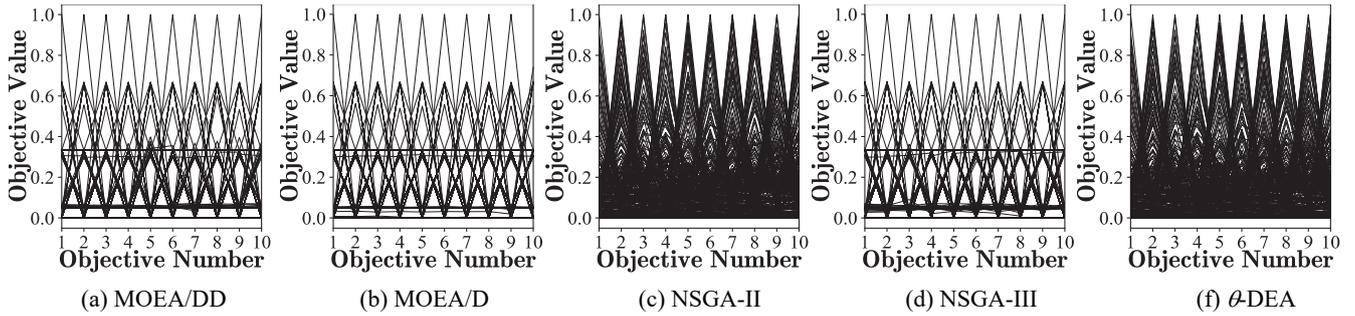


Figure E7. The obtained populations on the ten-objective type 03 test problem. In type 03 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, triangular, Minus-DTLZ, respectively.

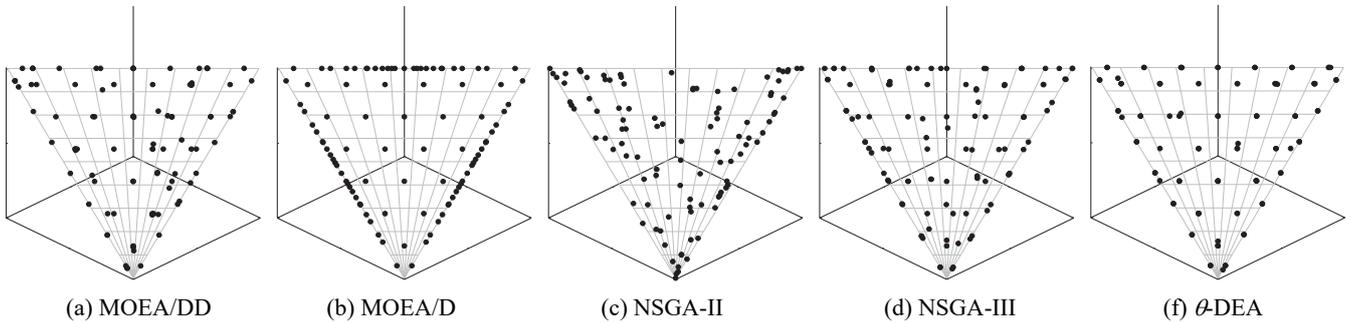


Figure E8. The obtained populations on the three-objective type 04 test problem. In type 04 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, DTLZ, respectively.

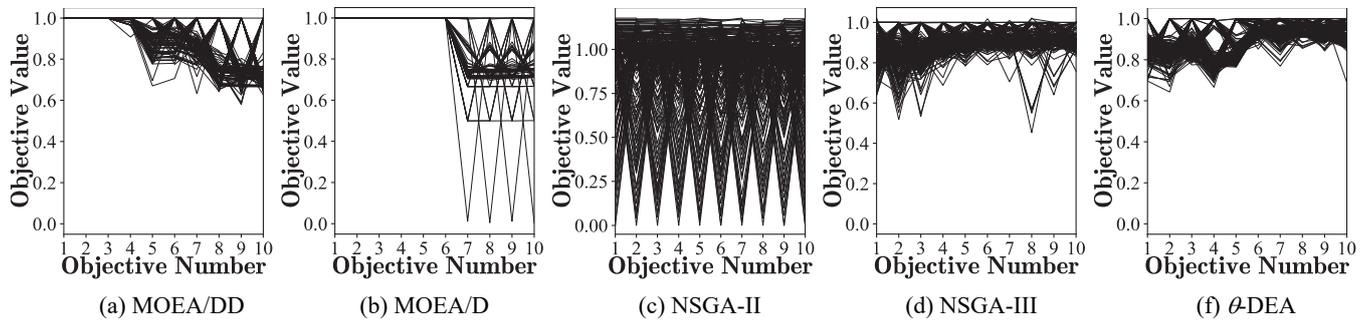


Figure E9. The obtained populations on the ten-objective type 04 test problem. In type 04 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, DTLZ, respectively.

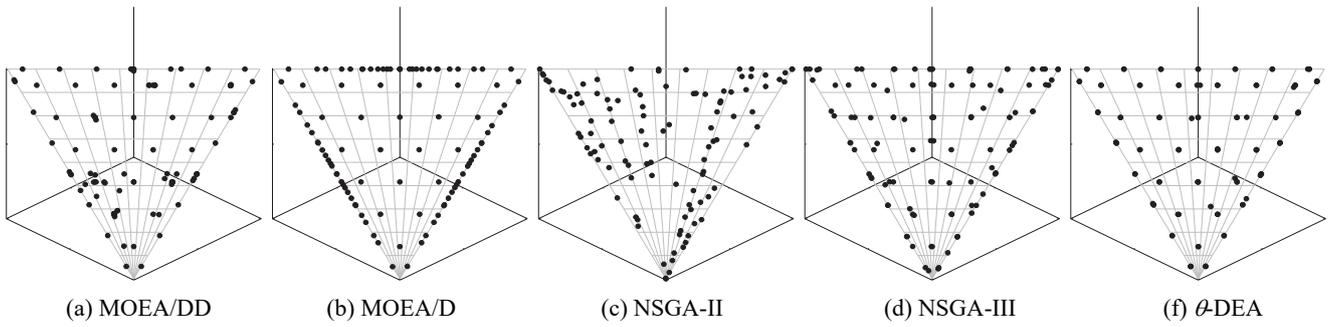


Figure E10. The obtained populations on the three-objective type 05 test problem. In type 05 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, WFG, respectively.

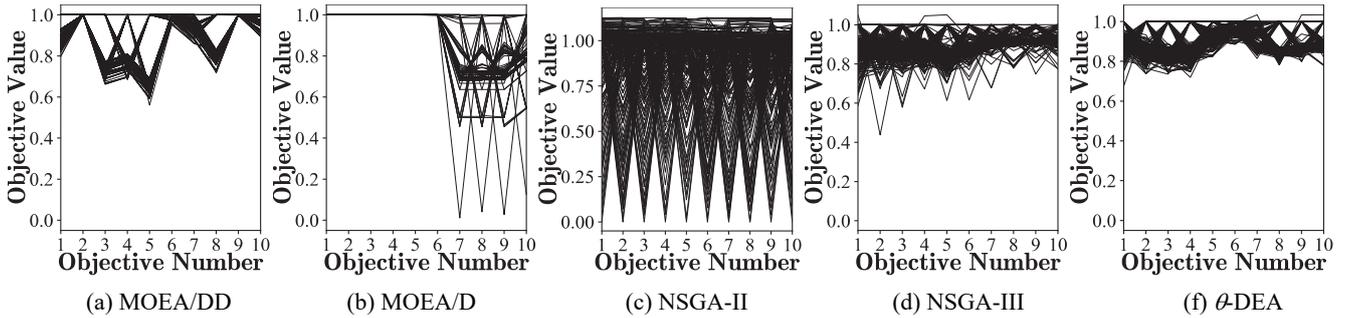


Figure E11. The obtained populations on the ten-objective type 05 test problem. In type 05 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, WFG, respectively.

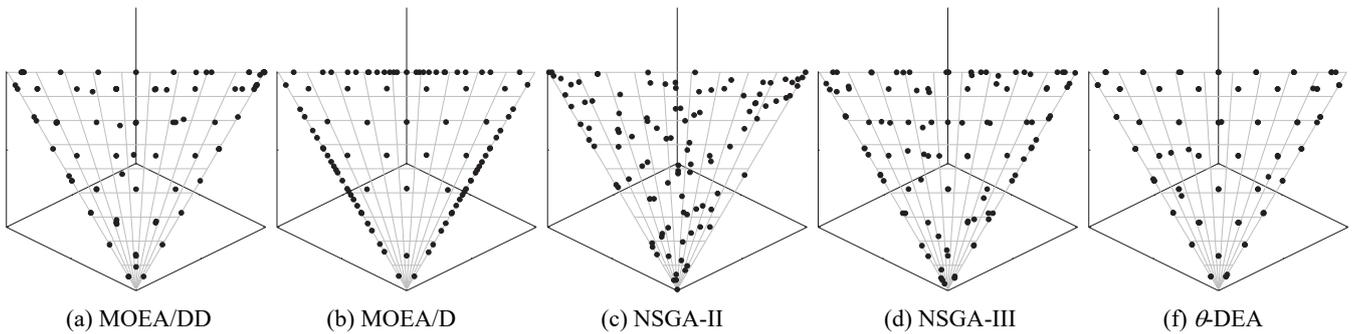


Figure E12. The obtained populations on the three-objective type 06 test problem. In type 06 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, Minus-DTLZ, respectively.

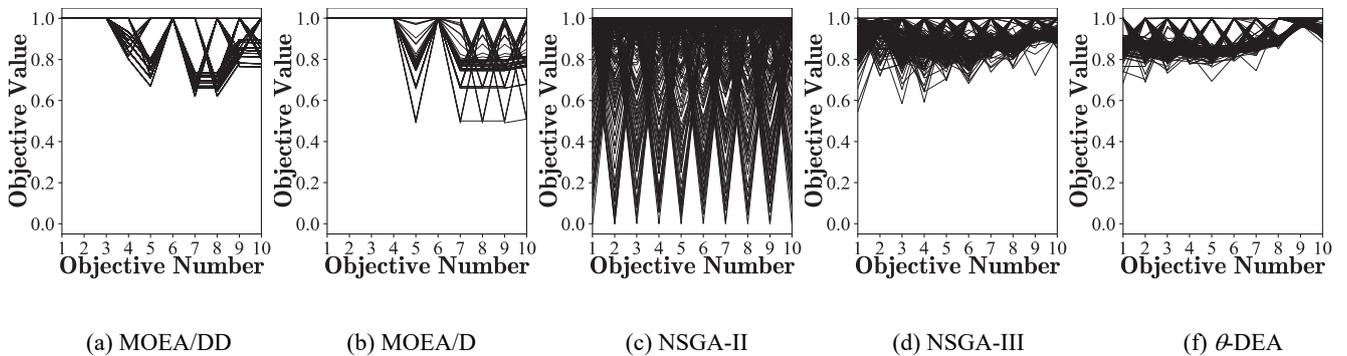


Figure E13. The obtained populations on the ten-objective type 06 test problem. In type 06 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are linear, inverted triangular, Minus-DTLZ, respectively.

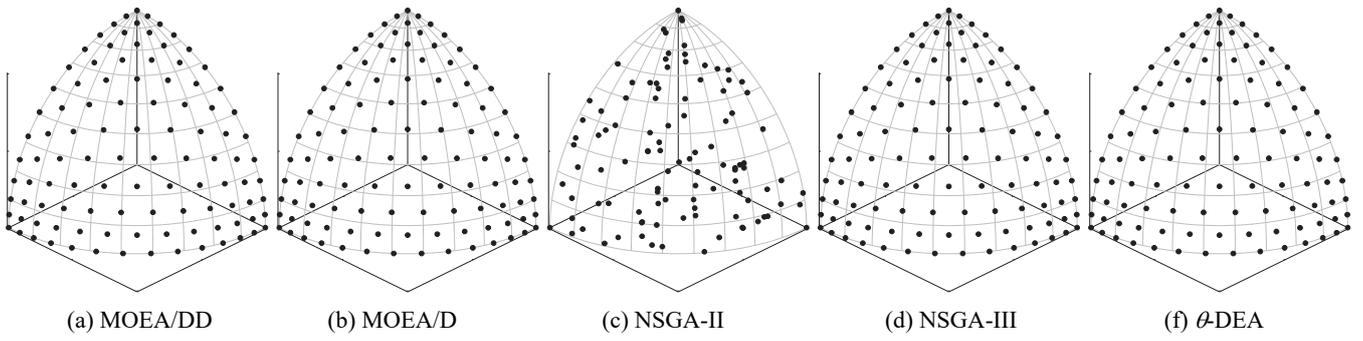


Figure E14. The obtained populations on the three-objective type 07 test problem. In type 07 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, triangular, DTLZ, respectively.

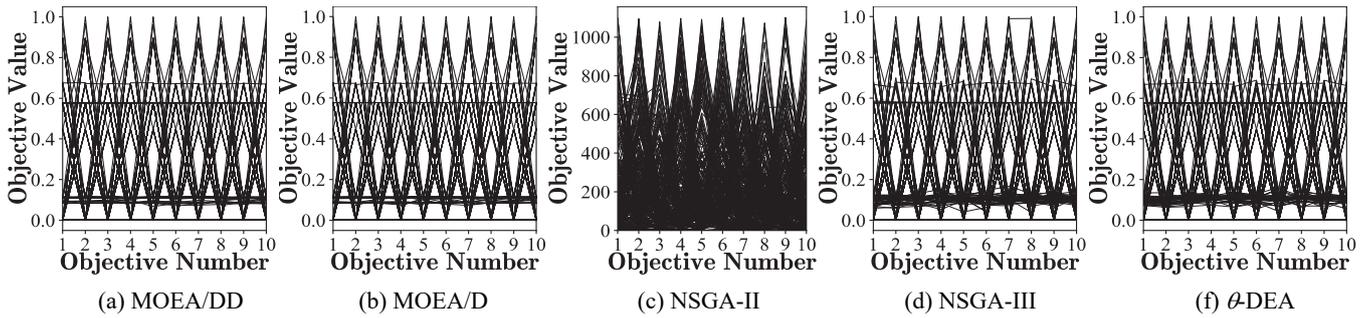


Figure E15. The obtained populations on the ten-objective type 07 test problem. In type 07 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, triangular, DTLZ, respectively.

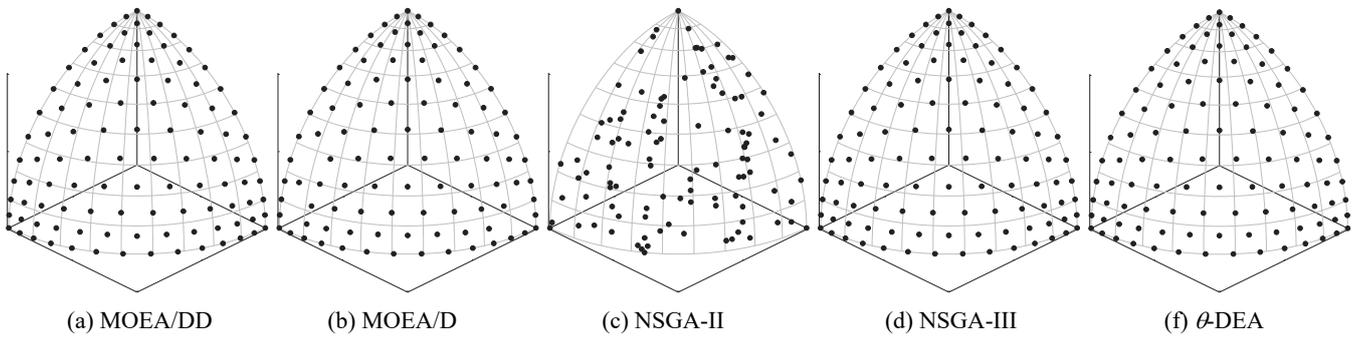


Figure E16. The obtained populations on the three-objective type 08 test problem. In type 08 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, WFG, respectively.

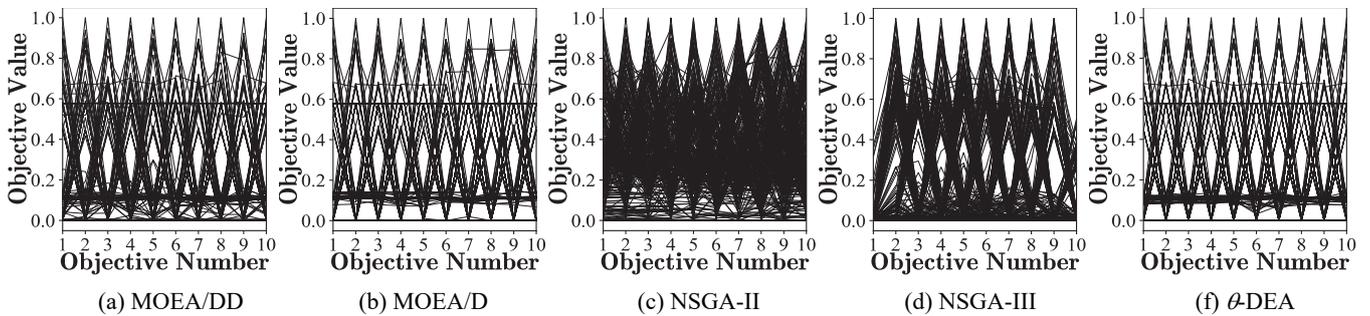


Figure E17. The obtained populations on the ten-objective type 08 test problem. In type 08 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, WFG, respectively.

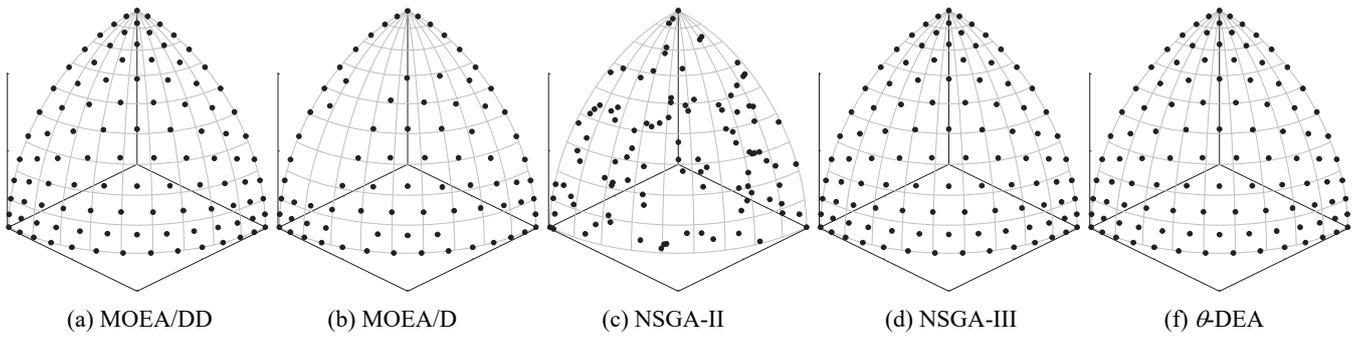


Figure E18. The obtained populations on the three-objective type 09 test problem. In type 09 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, triangular, Minus-DTLZ, respectively.

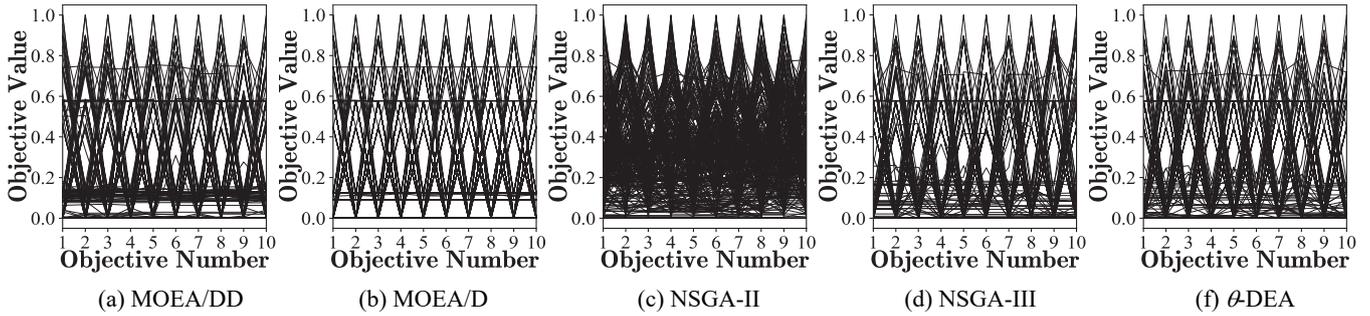


Figure E19. The obtained populations on the ten-objective type 09 test problem. In type 09 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, triangular, Minus-DTLZ, respectively.

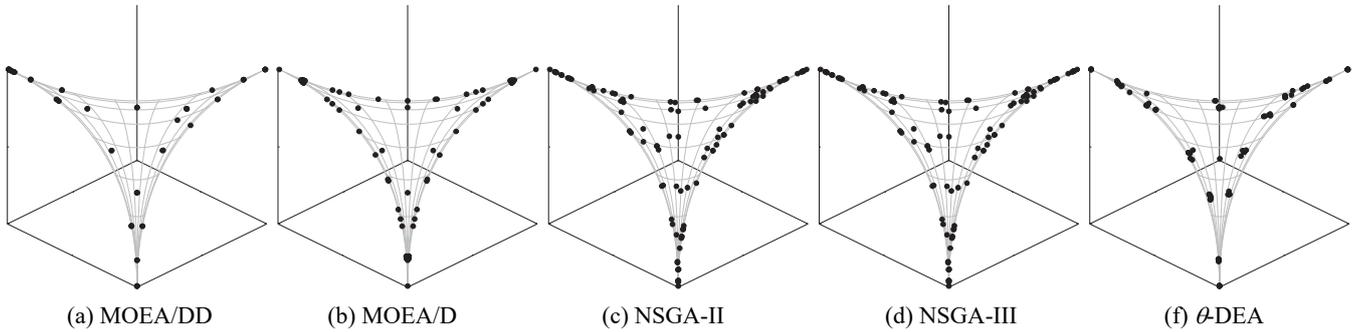


Figure E20. The obtained populations on the three-objective type 10 test problem. In type 10 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, DTLZ, respectively.

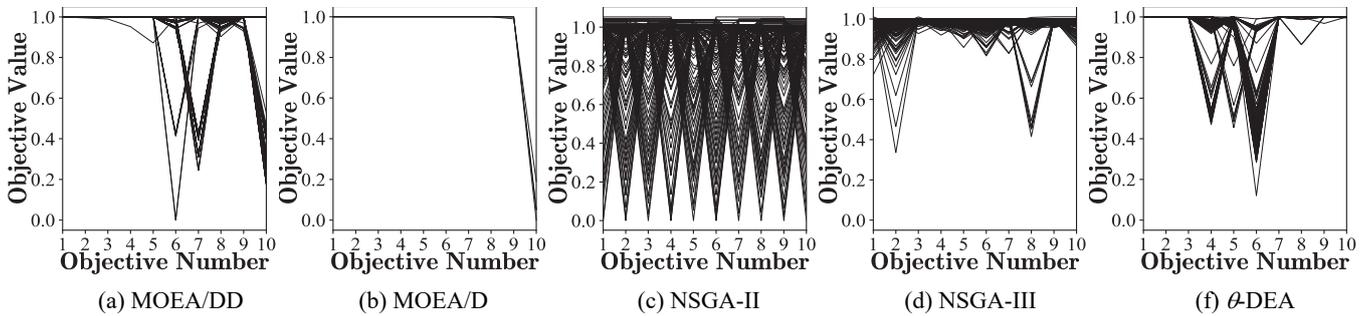


Figure E21. The obtained populations on the ten-objective type 10 test problem. In type 10 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, DTLZ, respectively.

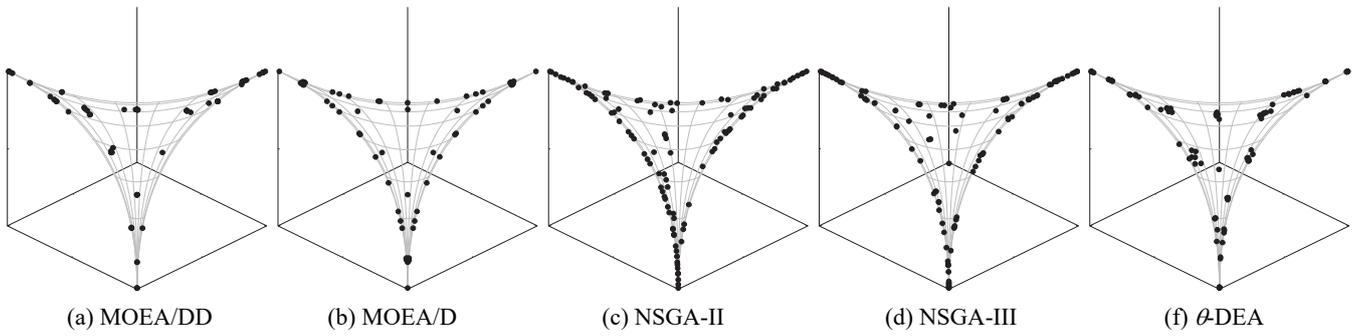


Figure E22. The obtained populations on the three-objective type 11 test problem. In type 11 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, WFG, respectively.

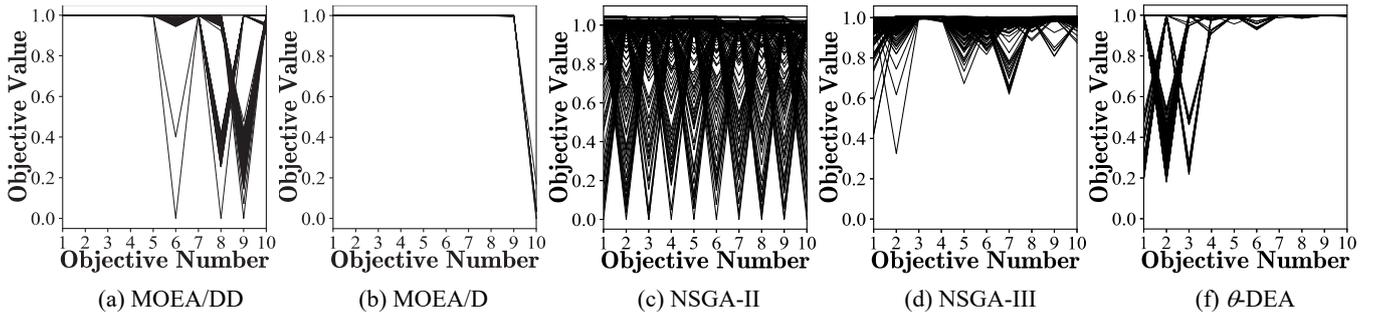


Figure E23. The obtained populations on the ten-objective type 11 test problem. In type 11 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, WFG, respectively.

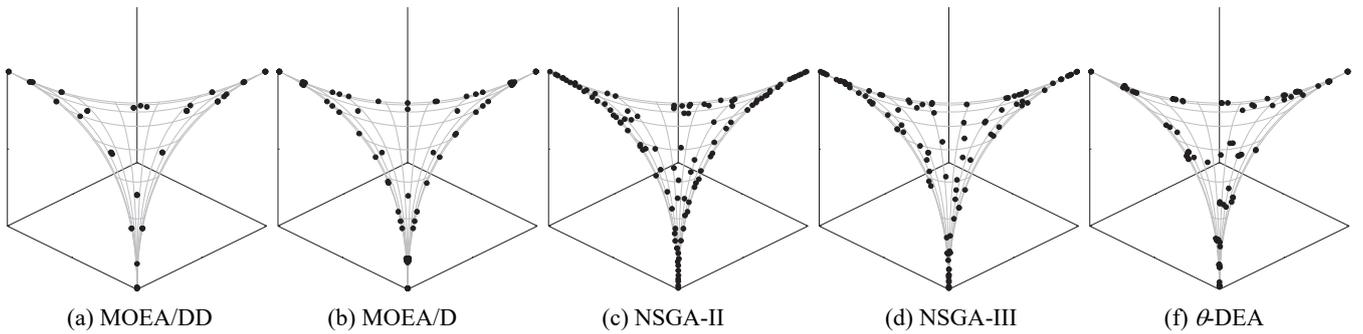


Figure E24. The obtained populations on the three-objective type 12 test problem. In type 12 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, Minus-DTLZ, respectively.

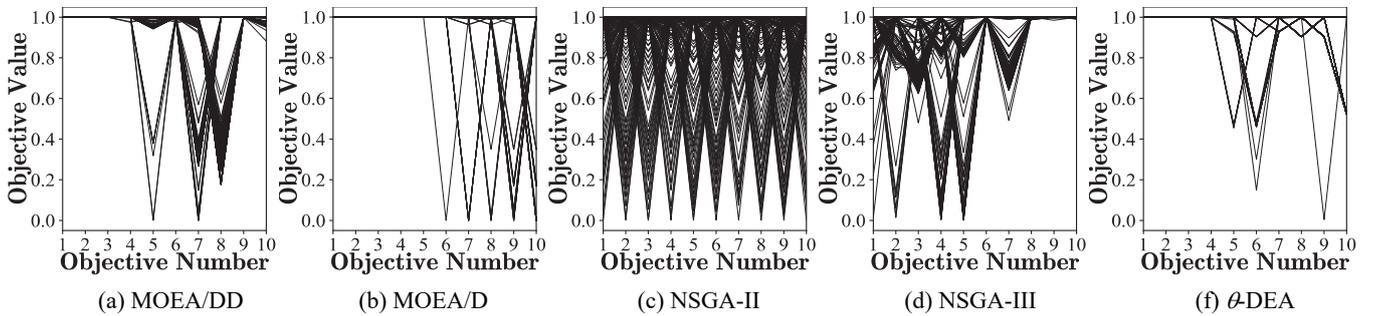


Figure E25. The obtained populations on the ten-objective type 12 test problem. In type 12 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are concave, inverted triangular, Minus-DTLZ, respectively.

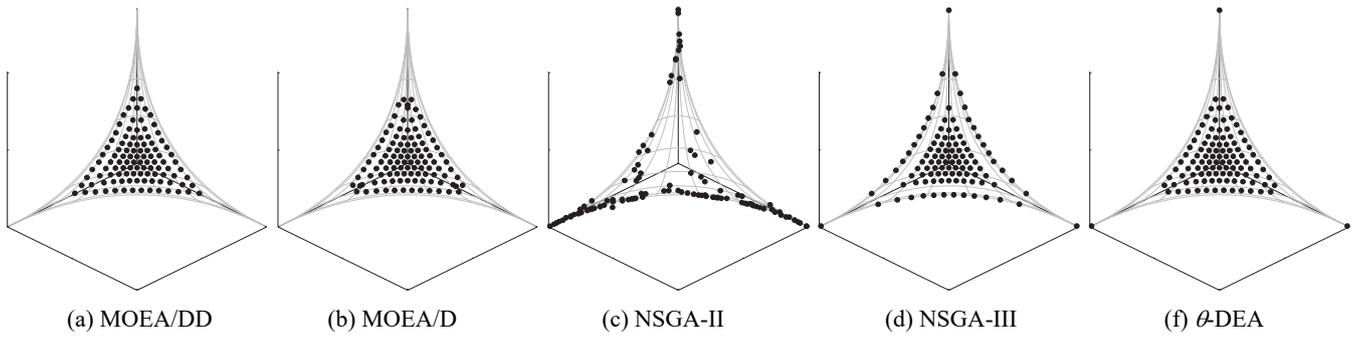


Figure E26. The obtained populations on the three-objective type 13 test problem. In type 13 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, DTLZ, respectively.

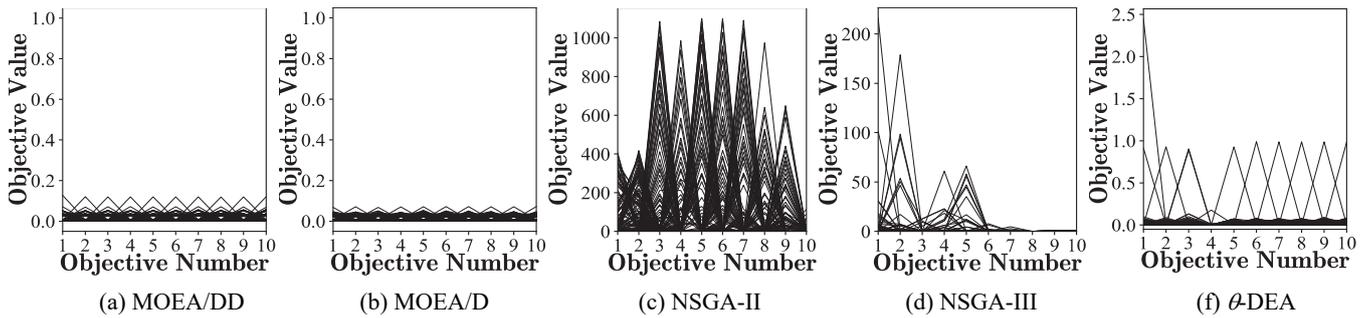


Figure E27. The obtained populations on the ten-objective type 13 test problem. In type 13 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, DTLZ, respectively.

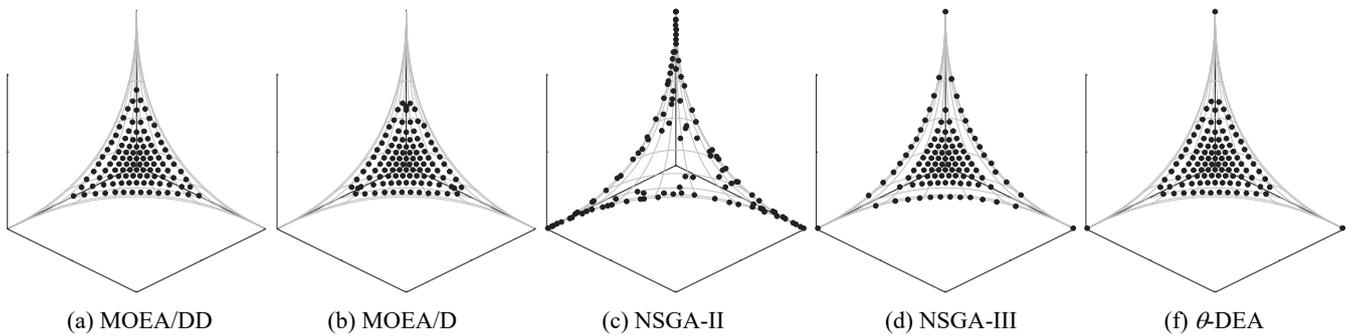


Figure E28. The obtained populations on the three-objective type 14 test problem. In type 14 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, WFG, respectively.

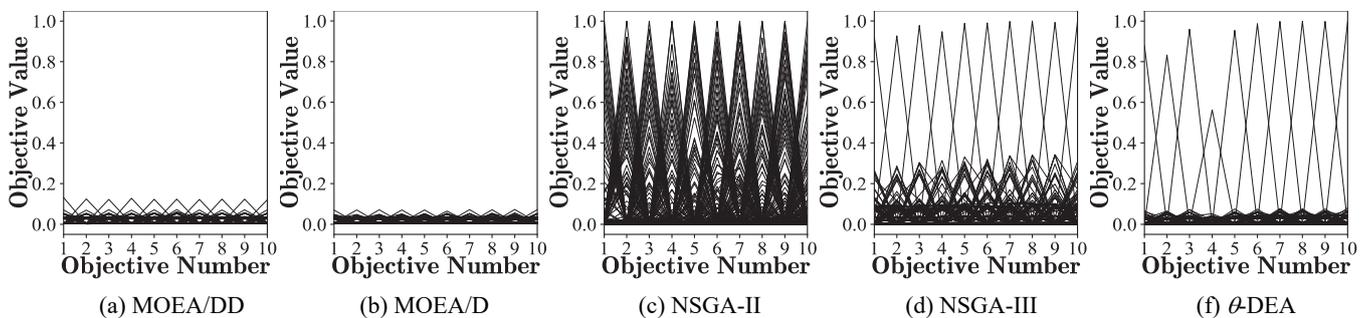


Figure E29. The obtained populations on the ten-objective type 14 test problem. In type 14 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, WFG, respectively.

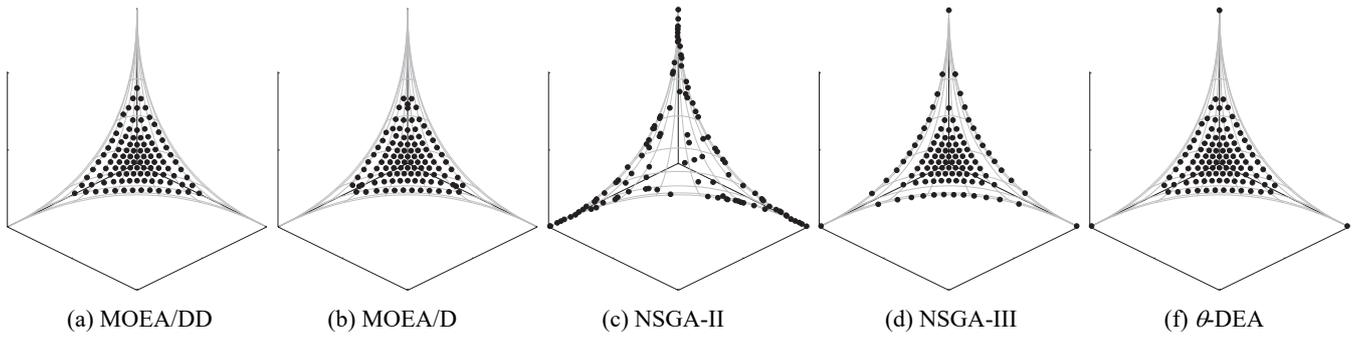


Figure E30. The obtained populations on the three-objective type 15 test problem. In type 15 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, Minus-DTLZ, respectively.

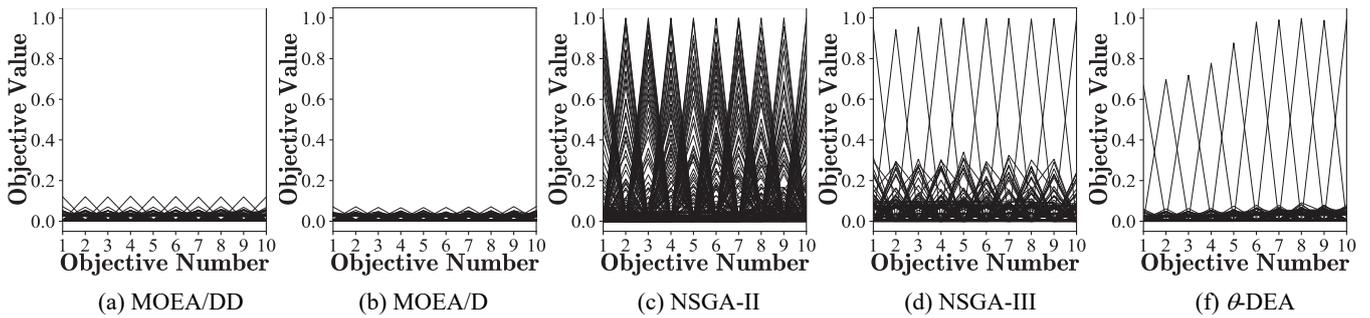


Figure E31. The obtained populations on the ten-objective type 15 test problem. In type 15 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, triangular, Minus-DTLZ, respectively.

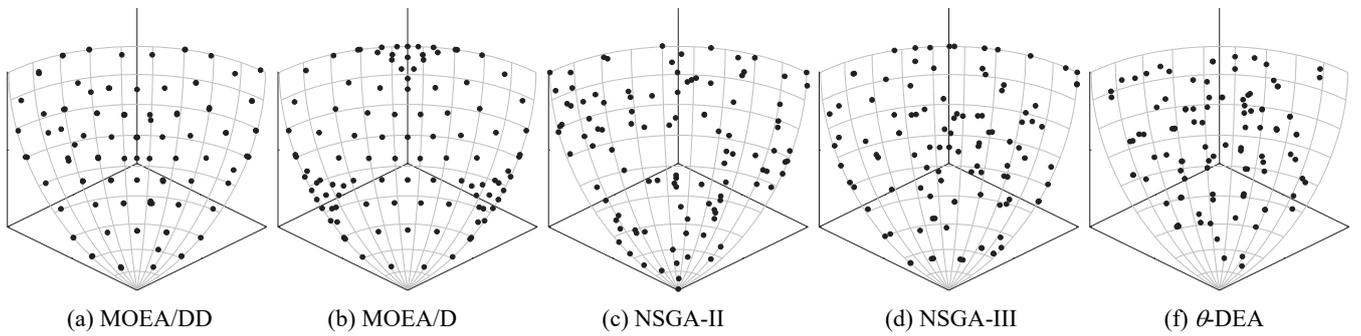


Figure E32. The obtained populations on the three-objective type 16 test problem. In type 16 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, DTLZ, respectively.

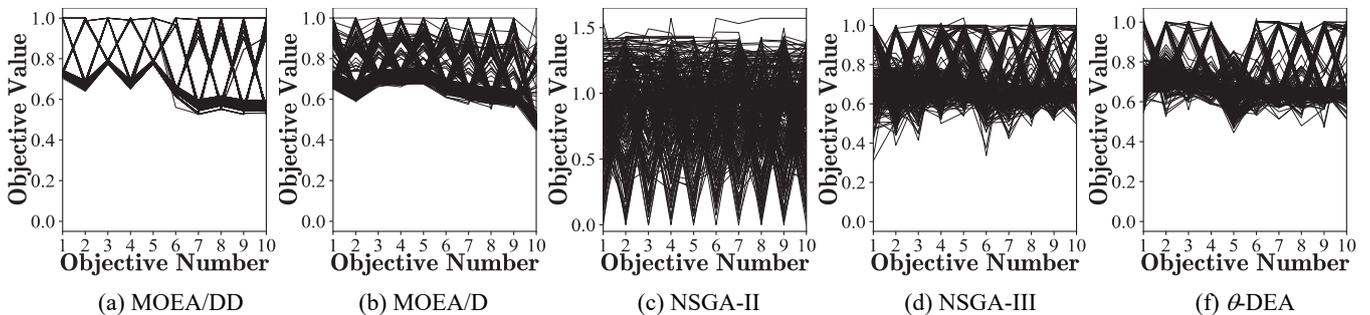


Figure E33. The obtained populations on the ten-objective type 16 test problem. In type 16 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, DTLZ, respectively.

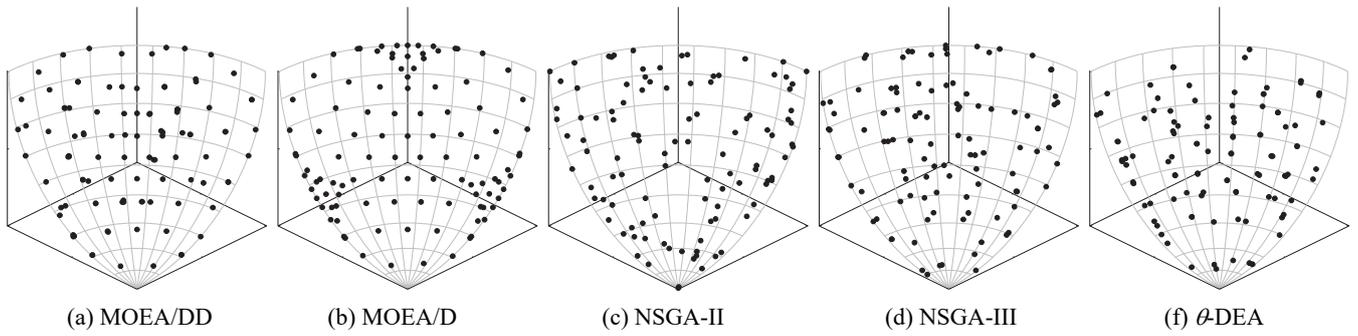


Figure E34. The obtained populations on the three-objective type 17 test problem. In type 17 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, WFG, respectively.

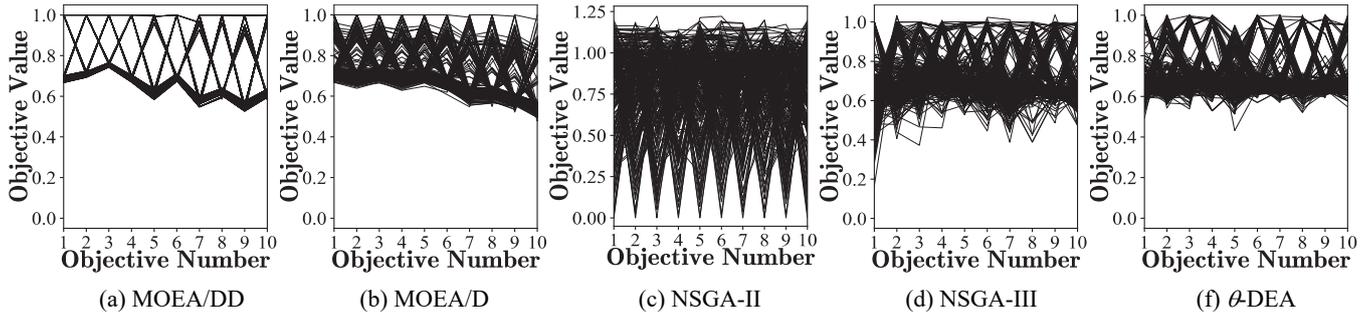


Figure E35. The obtained populations on the ten-objective type 17 test problem. In type 17 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, WFG, respectively.

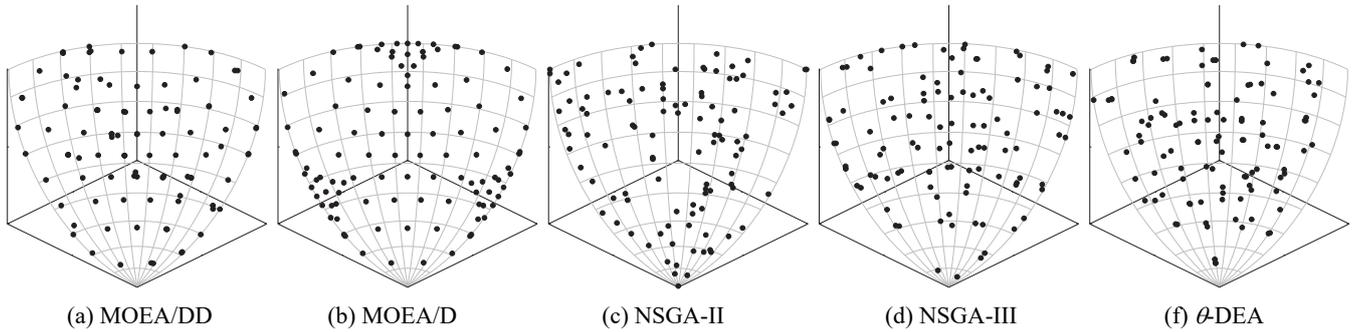


Figure E36. The obtained populations on the three-objective type 18 test problem. In type 18 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, Minus-DTLZ, respectively.

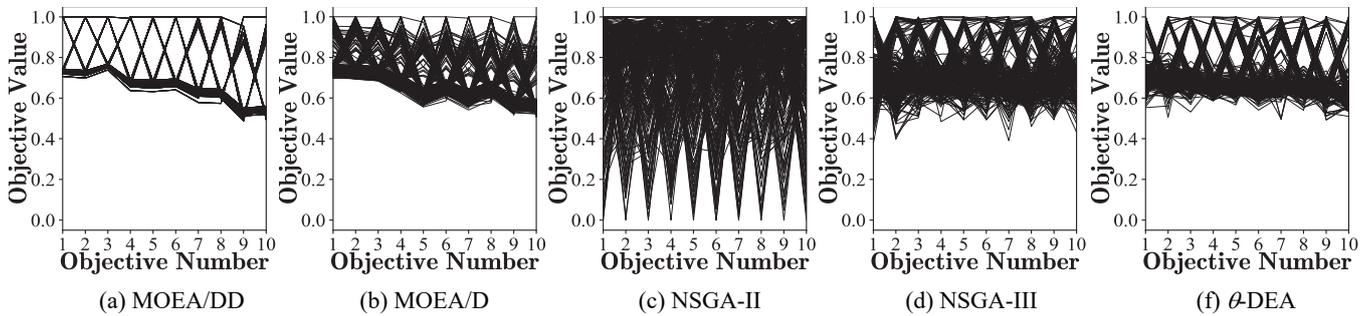


Figure E37. The obtained populations on the ten-objective type 18 test problem. In type 18 test problem, the curvature of the Pareto front, the shape of the Pareto front, the kind of feasible regions are convex, inverted triangular, Minus-DTLZ, respectively.