H. Ishibuchi, N. Tsukamoto, and Y. Nojima, "Evolutionary many-objective optimization: A short review," *Proc. of 2008 IEEE Congress on Evolutionary Computation*, pp. 2424-2431, Hong Kong, June 1-6, 2008.

Evolutionary Many-Objective Optimization: A Short Review

Hisao Ishibuchi, Noritaka Tsukamoto, and Yusuke Nojima

***Abstract*— Whereas evolutionary multiobjective optimization (EMO) algorithms have successfully been used in a wide range of real-world application tasks, difficulties in their scalability to many-objective problems have also been reported. In this paper, first we demonstrate those difficulties through computational experiments. Then we review some approaches proposed in the literature for the scalability improvement of EMO algorithms. Finally we suggest future research directions in evolutionary many-objective optimization.**

I. Introduction

EVOLUTIONARY multiobjective optimization (EMO) is one of the most active research areas in the field of evolutionary computation. A number of EMO algorithms have been proposed and successfully used in a wide range of real-world application tasks [1]-[4]. EMO algorithms usually work very well on two-objective problems. Their search ability is, however, severely deteriorated by the increase in the number of objectives. Multiobjective problems with four or more objectives are often referred to as many-objective problems. When we apply a well-known and frequently-used Pareto dominance-based EMO algorithm to such a many-objective problem, we may encounter a number of serious difficulties such as:

1. Deterioration of the search ability of Pareto dominance-based EMO algorithms such as SPEA [5] and NSGA-II [6]. When the number of objectives increases, almost all solutions in each population become non-dominated. This severely weakens the Pareto dominance-based selection pressure toward the Pareto front. That is, the convergence property of EMO algorithms is severely deteriorated.
2. Exponential increase in the number of solutions required for approximating the entire Pareto front. The goal of EMO algorithms is to find a set of non-dominated solutions that well approximates the entire Pareto front. Since the Pareto front is a hyper-surface in the objective space, the number of solutions required for its approximation exponentially increases with the dimensionality of the objective space (i.e., with the number of objectives). That is, we may need thousands of non-dominated solutions to approximate the entire Pareto front of a many-objective problem.
3. Difficulty of the visualization of solutions. It is usually assumed that the choice of a final solution from a set of obtained non-dominated solutions is done by a decision maker based on his/her preference. The increase in the number of objectives makes the visualization of obtained non-dominated solutions very difficult. This means that the choice of a final solution becomes very difficult in many-objective optimization.

The first difficulty (i.e., the deterioration of the search ability of EMO algorithms by the increase in the number of objectives) has been pointed out in a number of studies (for early studies, see [7], [8]). The deterioration of the search ability was clearly demonstrated through the comparison with multiple runs of single-objective optimizers in [9]-[11].

A straightforward idea for the scalability improvement of EMO algorithms to many-objective problems is to increase the selection pressure toward the Pareto front. One approach based on this idea is to modify Pareto dominance in order to decrease the number of non-dominated solutions in each population [12]. Another approach is to assign different ranks to non-dominated solutions [13]-[17].

Another idea for the scalability improvement is the use of different fitness evaluation mechanisms (instead of Pareto dominance). One approach based on this idea is the use of indicator-based evolutionary algorithms where indicator functions such as hypervolume are used to evaluate each solution [18], [19]. Another approach is to use a number of different scalarizing functions for fitness evaluation [9], [10], [20]-[22].

The second difficulty (i.e., the exponential increase in the number of non-dominated solutions that are necessary for the approximation of the Pareto front) has often been tackled by incorporating preference information in EMO algorithms [23]-[25]. Preference information is used to concentrate on a small region of the Pareto front while EMO algorithms are used to find multiple non-dominated solutions in such a small region of the Pareto front.

A direct approach to the handling of the third difficulty (the difficulty of the visualization of solutions) is to decrease the number of objectives [26]-[29]. Of course, dimensionality reduction (i.e., objective selection) can remedy not only the third difficulty but also the other difficulties. Visualization techniques of non-dominated solutions with many objectives have been proposed in the literature [30]-[32] where objective vectors are mapped into a low-dimensional space for their visualization. A number of visualization techniques of high-dimensional objective vectors have also been proposed in the field of multiple criteria decision making (MCDM [33]).

In this paper, we briefly explain the above-mentioned approaches to the handling of many-objective problems by evolutionary algorithms. We do not intend to give an exhaustive review of studies on evolutionary many-objective optimization. Our intention is to explain some representative

This work was partially supported by Grant-in-Aid for Scientific Research (B): KAKENHI (17300075).

Hisao Ishibuchi, Noritaka Tsukamoto, and Yusuke Nojima are with the Department of Computer Science and Intelligent Systems, Graduate School of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531, Japan (hisaoi@cs.osakafu-u.ac.jp, nori@ci.cs.osakafu-u.ac.jp, nojima@cs.osakafu-u.ac.jp).

approaches in order to understand research issues in the area of evolutionary many-objective optimization. This paper is organized as follows. First, we discuss the scalability of EMO algorithms to many-objective problems by examining the behavior of NSGA-II on multiobjective knapsack problems with 2, 4, 6, 8 objectives in Section II. Then, we explain the above-mentioned approaches to the scalability improvement in Section III where many-objective test problems are also mentioned. Finally, we conclude this paper by suggesting some future research directions in Section IV.

II. Illustration of Difficulties of EMO Algorithms

A. *Multiobjective Optimization Problems*

In general, a k-objective maximization problem is written as follows:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ ...,\ f_k(\mathbf{x}))\,, \qquad (1)$$

$$\text{subject to } \mathbf{x} \in \mathbf{X}\,, \qquad (2)$$

where $\mathbf{f}(\mathbf{x})$ is the k-dimensional objective vector, $f_i(\mathbf{x})$ is the i-th objective to be maximized, $\mathbf{x}$ is the decision vector, and $\mathbf{X}$ is the feasible region.

As test problems, we used 500-item knapsack problems with 2, 4, 6, 8 objectives. These problems are denoted as 2-500, 4-500, 6-500 and 8-500 problems in this paper. Our 2-500 and 4-500 problems are the same as those in Zitzler and Thiele [5]. On the other hand, we generated 6-500 and 8-500 problems in the same manner as the generation procedure of the 2-500 and 4-500 problems in [5].

B. *Pareto Dominance Relation*

Let $\mathbf{y}$ and $\mathbf{z}$ be two feasible solutions of the k-objective maximization problem in (1)-(2). If the following conditions hold, $\mathbf{z}$ can be viewed as being better than $\mathbf{y}$:

$$\forall i : f_i(\mathbf{y}) \le f_i(\mathbf{z}) \ \text{ and } \ \exists j : f_j(\mathbf{y}) < f_j(\mathbf{z})\,. \qquad (3)$$

In this case, we say that $\mathbf{z}$ dominates $\mathbf{y}$ (equivalently $\mathbf{y}$ is dominated by $\mathbf{z}$: $\mathbf{z}$ is better than $\mathbf{y}$).

When $\mathbf{y}$ is not dominated by any other feasible solutions, $\mathbf{y}$ is referred to as a Pareto-optimal solution of the k-objective maximization problem in (1)-(2). The set of all Pareto-optimal solutions forms the tradeoff surface in the objective space. This tradeoff surface is referred to as the Pareto front. EMO algorithms are designed to search for a set of well-distributed non-dominated solutions that approximates the entire Pareto front very well.

In order to examine the relation between the percentage of non-dominated solutions in a population and the number of objectives, we randomly generated 200 objective vectors in the k-dimensional unit-hypercube $[0, 1]^k$ for k = 2, 4, ..., 20. Among the generated 200 objective vectors for each k, we calculated the percentage of non-dominated vectors. The average percentage over 10 runs for each k is shown in Fig. 1. We can see from Fig. 1 that almost all objective vectors are non-dominated when k is larger than 10. Similar figures have already been depicted in several studies to illustrate the difficulty of many-objective problems (e.g., [1], [17]).

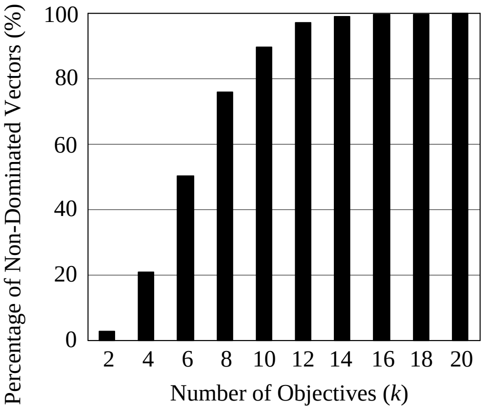


Fig. 1. Average percentage of non-dominated vectors among 200 vectors that are randomly generated in the k-dimensional unit hypercube $[0, 1]^k$.

C. *Settings of Computational Experiments*

We applied NSGA-II [6] to our four test problems using the following parameter settings:

Population size: 100,
Crossover probability: 0.8 (Uniform crossover),
Mutation probability: 1/500 (Bit-flip mutation),
Stopping conditions: 100,000 generations.

We executed NSGA-II for a large number of generations (i.e., 100,000 generations) to examine its long-term behavior. NSGA-II was applied to each test problem ten times. The average behavior of NSGA-II was examined for each test problem using the following four measures:

Number of non-dominated solutions

We counted the number of non-dominated solutions in the merged population (i.e., the union of the current population and its offspring population) with 200 solutions in each generation. When the number of non-dominated solutions is equal to or larger than 100, all solutions in the next population are non-dominated with each other. In this case, Pareto sorting has no effect on parent selection in the next population. That is, there exists no selection pressure toward the Pareto front in the parent selection phase.

Maximum sum of the objective values: MaxSum

In each generation (i.e., for the current population with 100 solutions), we calculated the maximum sum of the objective values as follows:

$$\text{MaxSum}(\Psi) = \max_{\mathbf{x}\in\Psi} \sum_{i=1}^{k} f_i(\mathbf{x})\,, \qquad (4)$$

where Ψ denotes the current population, and k is the number of the given objectives in each test problem (k = 2, 4, 6, 8). This measure evaluates the convergence of solutions toward the Pareto front around its center region.

Sum of the maximum objective values: SumMax

The sum of the maximum objective value of each objective was calculated in each generation as follows:

$$\text{SumMax}(\Psi) = \sum_{i=1}^{k} \max_{\mathbf{x}\in\Psi} f_i(\mathbf{x})\,. \qquad (5)$$

This measure evaluates the convergence of solutions toward the Pareto front around its k edges.

Sum of the ranges of the objective values: Range

The sum of the range of objective values of each objective was calculated in each generation as follows:

$$\text{Range}(\Psi) = \sum_{i=1}^{k} [\max_{\mathbf{x}\in\Psi}\{f_i(\mathbf{x})\} - \min_{\mathbf{x}\in\Psi}\{f_i(\mathbf{x})\}]. \quad (6)$$

This measure evaluates the diversity of solutions in the objective space in each generation.

D. *Experimental Results of NSGA-II*

In Fig. 2, we show the average number of non-dominated solutions at each generation for each test problem. More than 100 non-dominated solutions were almost always included in the merged population except for very early generations in Fig. 2. The number of non-dominated solutions increased with the number of objectives. Similar observations have been reported in Sato et al. [12].

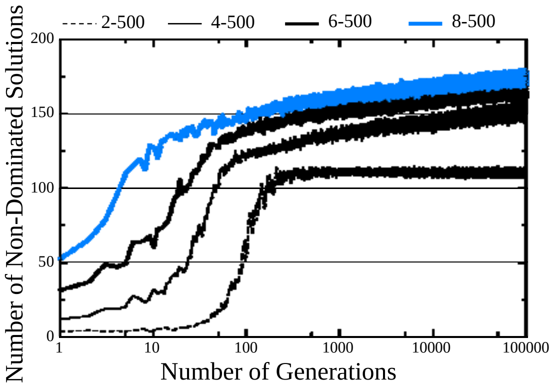


Fig. 2. Average number of non-dominated solutions in the merged population with 200 solutions at each generation. NSGA-II chooses the best 100 solutions from the merged population to form the next population.

Whereas the percentage of non-dominated vectors was about 20% among 200 randomly generated four-dimensional vectors in Fig. 1, it was much larger than 50% in the merged population during the execution of NSGA-II on the 4-500 problem except for very early generations (e.g., except for the first 50 generations) in Fig. 2. This means that the selection pressure toward the Pareto front was very weak even in the case of the four-objective problem (i.e., 4-500 problem).

We show the convergence property of NSGA-II using the MaxSum measure in Fig. 3. Experimental results in Fig. 3 are normalized so that the average result of initial populations over ten runs becomes 100 for each test problem. We use the same normalization procedure for all performance measures throughout this paper (i.e., we always use the average result of initial populations as the baseline value 100). We can see from Fig. 3 that the convergence to the center region of the Pareto front was slowed down by the increase in the number of objectives. This is because the selection pressure toward the Pareto front by the Pareto sorting was severely weakened by the increase in the number of non-dominated solutions (see Fig. 2). One interesting observation is that the MaxSum measure first increased then decreased during the execution of NSGA-II for the 4-500 and 6-500 problems. We can not observe such a non-monotonic convergence behavior for the 2-500 problem. We can also observe that the convergence toward the Pareto front was very slow in the case of the 8-500 problem in Fig. 3.

From the comparison between Fig. 2 and Fig. 3, we can see that NSGA-II has a strong convergence property only when the number of non-dominated solutions is less than 50% of the merged population (i.e., less than 100 in Fig. 2).

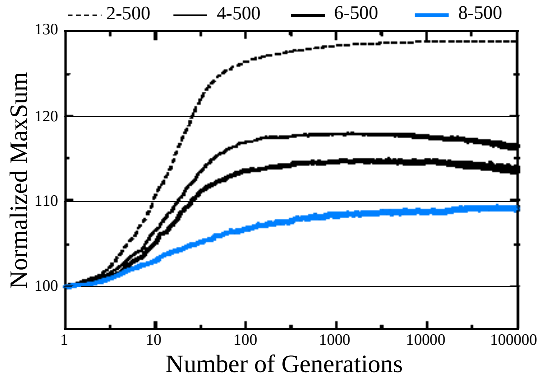


Fig. 3. Convergence around the center region of the Pareto front by the original NSGA-II algorithm.

In Fig. 4, we show the convergence behavior to the Pareto front around its edges using the SumMax measure. The average value of the SumMax measure was gradually improved during the execution of NSGA-II over a large number of generations in Fig. 4. This observation suggests the difficulty in finding a set of non-dominated solutions that covers the entire Pareto front within a small number of generations. The same observation was reported in [34].

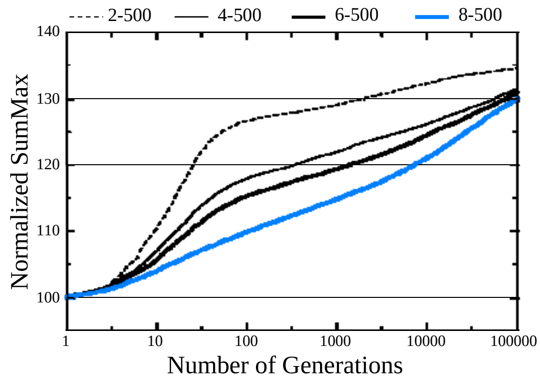


Fig. 4. Convergence around the edges of the Pareto front by the original NSGA-II algorithm.

In Fig. 5, we show the diversity of solutions in each generation using the Range measure. From Fig. 2 and Fig. 5, we can see that NSGA-II started to improve the diversity after the number of non-dominated solutions exceeded 100. When the number of non-dominated solutions was less than 100 (i.e., in early generations), the diversity of solutions decreased in Fig. 5. At the same time, the convergence to the Pareto front around its center region was rapidly improved in Fig. 3.

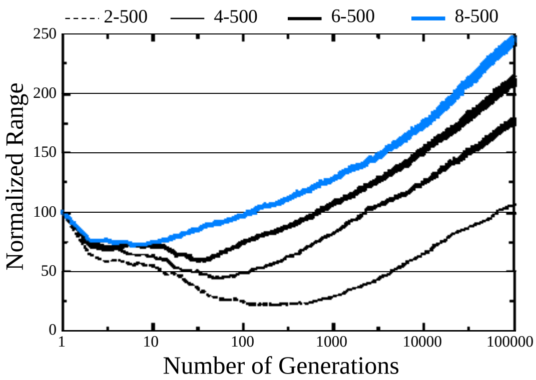


Fig. 5. Diversity of solutions in the case of the original NSGA-II algorithm.

E. Minor Changes for Many-Objective Problems

Before explaining scalability improvement approaches in the next section, we examine the effectiveness of two minor changes of NSGA-II for many-objective problems. One is the assignment of a zero distance (instead of an infinity distance) to extreme solutions with maximum or minimum objective values as the crowding distance, which was suggested in [18].

Experimental results are shown in Fig. 6 and Fig. 7. From the comparison of Fig. 6 with Fig. 3, we can see that the convergence was somewhat improved by the modification of the crowding distance for many-objective problems (e.g., for the 8-500 problem). This improvement was realized at the cost of diversity (compare Fig. 7 with Fig. 5).

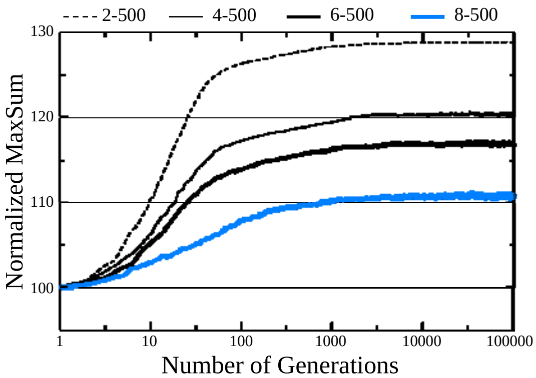


Fig. 6. Convergence by NSGA-II with the modified crowding distance.

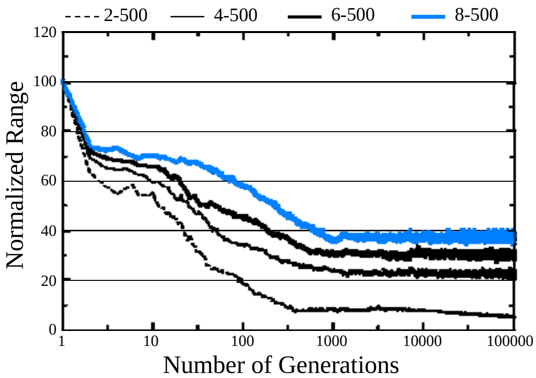


Fig. 7. Diversity by NSGA-II with the modified crowding distance.

Another minor change is the modification of the objective functions in order to increase the selection pressure toward the Pareto front, which was suggested in [21]. We examined the following modification in this paper:

$$g_i(x) = f_i(x) + \beta \times \sum_{j=1}^{k} f_j(x),\ i = 1, 2, ..., k, \qquad (7)$$

where β is a prespecified constant ($\beta = 1$ in this paper).

Experimental results are shown in Fig. 8 and Fig. 9. In Fig. 8, we can observe a clear improvement in the convergence property of NSGA-II by the modification of the objective functions. Actually Fig. 8 is much better than Fig. 3 and better than Fig. 6 especially for many-objective problems. As in the case of the modification of the crowding distance in Fig. 7, such a convergence improvement was realized at the cost of diversity (compare Fig. 9 with Fig. 5).

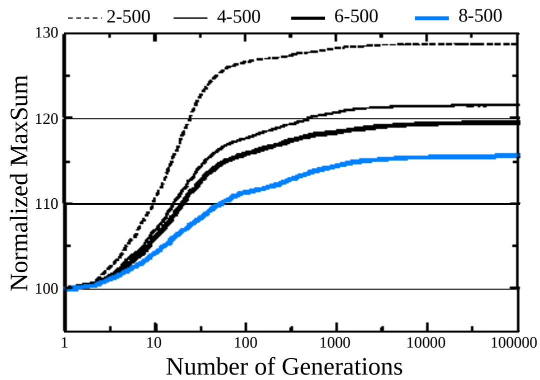


Fig. 8. Convergence by NSGA-II with the modified objective functions.

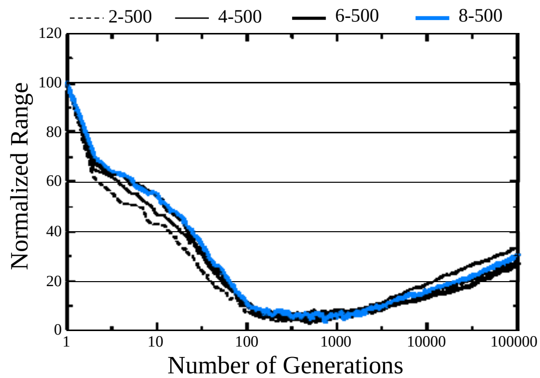


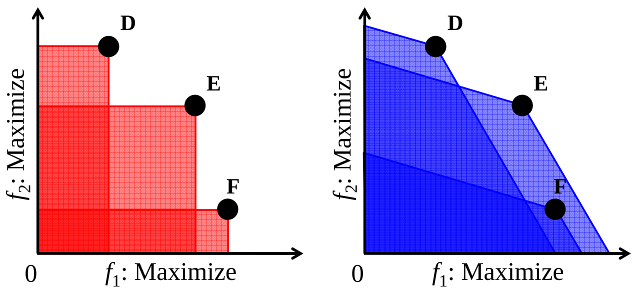
Fig. 9. Diversity by NSGA-II with the modified objective functions.

III. APPROACHES TO MANY-OBJECTIVE OPTIMIZATION

In this section, we explain the scalability improvement approaches mentioned in Section I. We also describe many-objective test problems.

A. Modification of Pareto Dominance

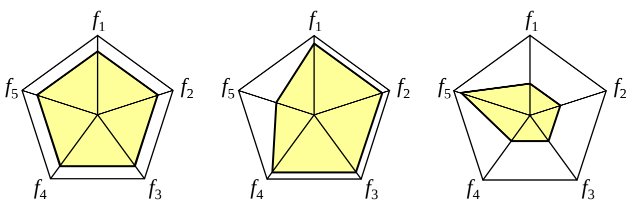
Sato et al. [12] demonstrated that the use of a modified dominance (instead of the standard Pareto dominance) clearly improved the performance of NSGA-II for many-objective problems. In order to increase the selection pressure toward the Pareto front by the Pareto sorting in NSGA-II, they modified Pareto dominance as shown in Fig. 10 where the dominated region by each solution is shaded. When we use the standard Pareto dominance in Fig. 10 (a), all the three solutions are non-dominated with each other. On the other hand, solution F is dominated by solution E if we use the modified dominance in Fig. 10 (b). In this manner, the selection pressure toward the Pareto front can be strengthened because the number of non-dominated solutions in each population is decreased by the use of the modified dominance. The extent of the modification of Pareto dominance (i.e., the angle of the dominated region in Fig. 10 (b)) should be adjusted to the number of objectives. Roughly speaking, the increase in the number of objectives requires a wider angle of the dominated region in Fig. 10 (b) as suggested by [12].



(a) Original Pareto dominance. (b) Modified dominance.

Fig. 10. Illustration of the modification of Pareto dominance in [12].

Modification of Pareto dominance in EMO algorithms has often been discussed in the EMO community. One basic idea is as follows: A large deterioration in one objective can not be accepted for only a small improvement in another objective. For example, let us consider the three solutions of a five-objective maximization problem in Fig. 11 where each pentagon represents a five-dimensional objective vector. Roughly speaking, larger pentagons mean better solutions because we assume maximization. Whereas all the three solutions in Fig. 11 are non-dominated, one may think that solution C seems to be inferior to the other solutions. This is because solution C has much worse objective values with respect to the first four objectives (i.e., f_1, f_2, f_3 and f_4) than the other solutions whereas it has a better objective value only for f_5. On the other hand, one may think that solution A is a good solution because it has good objective values for all the five objectives. Contrary to this intuition, NSGA-II handles solution C as being better than solution A based on their crowding distances (because C has an infinity distance).



(a) Solution A. (b) Solution B. (c) Solution C.

Fig. 11. Three non-dominated objective vectors.

Similar ideas to Sato et al. [12] have been discussed in the literature. For example, the concept of α-dominance was proposed for archive maintenance by Ikeda et al. [35] where the angle of the dominance region was widened as in Fig. 10 (b). Branke et al. [36] also proposed a similar idea for guiding multiobjective evolution whereas they explained it as a preference incorporation method. Their idea was compared with a reference solution-based approach in [37].

As pointed out in Branke et al. [36], the modification of Pareto dominance has a similar effect on multiobjective evolution as the modification of objective functions such as (7). This means that the increase in the selection pressure toward the Pareto front by the modification of Pareto dominance leads to the decrease in the diversity of solutions as shown in Fig. 8 and Fig. 9 in the previous section. The same observations were reported in Sato et al. [12].

B. Introduction of Different Ranks

Drechsler et al. [13] proposed the use of a relation called *favour* to differentiate between non-dominated solutions for the handling of many-objective problems. They defined the relation *favour* based on the number of objectives for which one solution is better than the other. More specifically, a solution $\mathbf{z}$ is viewed as being better than another solution $\mathbf{y}$ under the relation *favour* when the following relation holds:

$$\left|\{j : f_j(\mathbf{z}) < f_j(\mathbf{y}), 1 \le j \le k\}\right| < \left|\{i : f_i(\mathbf{y}) < f_i(\mathbf{z}), 1 \le i \le k\}\right| . \quad (8)$$

If we apply this relation to the three non-dominated vectors in Fig. 11, solution B is viewed as the most preferred solution since it has better objective values than the other two with respect to the first four objectives (i.e., f_1, f_2, f_3 and f_4). On the other hand, solution C is viewed as the least preferred solution under the relation *favour*.

The relation *favour* was modified in Sülflow et al. [14] by taking into account not only the number of objectives for which one solution is better than the other but also the difference in objective values between the two solutions.

Various ranking methods were compared with each other in [15]-[17]. For example, Corne and Knowles [16] reported that the best results were obtained from a simple average ranking method than more complicated ranking schemes. In the average ranking method, first a rank for each objective is assigned to each solution based on the ranking of its objective value for the corresponding objective among non-dominated solutions in the current population. Thus each solution has k ranks, each of which is based on one of the k objectives. Then the average rank is calculated for each solution as its rank. In Kukkonen and Lampinen [17], the average and minimum ranking methods were examined. Köppen and Yoshida [15] examined more complicated ranking methods based on ε-dominance and fuzzy Pareto dominance.

As in the case of the modification of Pareto dominance, the introduction of different ranks to non-dominated solutions leads to the increase in the selection pressure toward the Pareto front and the decrease in the diversity of solutions. In some cases, the population converges to a few solutions (or a single solution) as reported in [16].

C. Use of Indicator Functions

A number of performance indicators have been proposed to measure the quality of non-dominated solution sets [38]-[40]. Since the performance of EMO algorithms is often evaluated by those indicators, it is a promising idea to directly optimize an indicator (e.g., hypervolume [41]) in EMO algorithms. This idea was used for archive maintenance in Knowles et al. [42]. A general framework of indicator-based evolutionary algorithms (IBEAs) was proposed by Zitzler and Künzli [43]. Several variants of IBEAs have been proposed in the literature [44]-[46]. In those variants of IBEAs, hypervolume was almost always used as an indicator.

Wagner et al. [18] reported good results by IBEAs for many-objective problems. Since IBEAs do not use Pareto dominance, their search ability is not severely deteriorated by the increase in the number of objectives. One difficulty in the application of IBEAs to many-objective problem is a large computation cost for hypervolume calculation. Ishibuchi et al. [19] proposed an iterative version of IBEAs to decrease the computation cost by searching for only a small number of representative solutions. Objective reduction in IBEAs was examined in Brockhoff and Zitzler [28] for the same purpose.

D. Use of Scalarizing Functions

The main advantage of the use of scalarizing functions for many-objective problems is their efficiency. For example, we can easily calculate weighted sums of multiple objectives even when the number of objectives is large. On the other hand, the computation time for hypervolume calculation exponentially increases with the number of objectives.

There exist two different classes of EMO algorithms that use scalarizing functions for many-objective problems. In one class, a large number of scalarizing functions are used for evaluating each solution [22], [47]. A rank of each solution is calculated for each scalarizing function. Thus each solution has multiple ranks. The number of those ranks is the same as that of scalarizing functions. The overall rank of each solution is calculated based on its multiple ranks.

In the other class, each solution is evaluated by a single scalarizing function [20], [21]. The point in this class of approaches is that a different scalarizing function (e.g., a weighted sum fitness function with a different weight vector) is used for evaluating each solution. The same idea is used in multiobjective genetic local search (MOGLS [48]-[50]).

E. Use of Preference Information

EMO algorithms are usually designed to search for a set of non-dominated solutions that approximates the entire Pareto front. Since the number of necessary solutions for a good approximation exponentially increases with the number of objectives, it is a good idea to focus on a specific region of the Pareto front using the decision maker's preference.

Fleming et al. [23] applied a preference articulation method [51] to many-objective problems where the focused region gradually becomes smaller during multiobjective evolution. Deb and Sundar [24] incorporated reference point-based preference information into NSGA-II. In their approach, the normalized distance to a reference point is taken into account to evaluate each solution after Pareto sorting. This means that the normalized distance is used as a secondary criterion instead of the crowding distance in NSGA-II. A number of non-dominated solutions are obtained around the reference point. Multiple reference points can be handled in [24].

Thiele et al. [25] used reference point-based preference information in IBEA. First a rough approximation of the Pareto front is obtained. Such a rough approximation is used by the decision maker to specify a reference point. Then IBEA searches for a number of non-dominated solutions around the reference point. The focus of the multiobjective search by IBEA can be controlled using a parameter value and a reference point. Usually the focused region gradually becomes smaller through the interaction with the decision maker. The use of preference information in [25] is based on a similar idea to weighted integration in [46].

In the above-mentioned methods, preference information is used to focus on a specific region in the high-dimensional objective space with many objectives while EMO algorithms are used to search for a number of non-dominated solutions in such a focused region. The use of preference information has been discussed in many studies in the EMO community (e.g., see [1], [2], [52], [53]). Much more studies on this issue can be found in the MCDM community [33].

F. Reduction of the Number of Objectives

If we can choose only a few important objectives, almost all difficulties in evolutionary many-objective optimization are eliminated. Deb and Saxena [26], [27] proposed an objective reduction method, which is based on principle component analysis. Their idea is to remove unnecessary objectives while maintaining the shape of the Pareto front in the reduced objective space. On the other hand, Brockhoff and Zitzler [28], [29] proposed a different idea where objective reduction is based on Pareto dominance. That is, an objective is removed when it does not change (or only slightly change) the Pareto dominance relation among solutions.

G. Visualization of Obtained Solutions

The main interest in the EMO community has been the search for a set of non-dominated solutions that approximates the Pareto front. Thus the choice of a final solution has not been discussed in many studies in the EMO community. Visualization of obtained non-dominated solutions, however, is a very important issue because a final solution should be chosen by the decision maker in any real-world applications.

There exist two classes of visualization methods. In one class, all objectives are used with no modifications. The pentagonal representation in Fig. 11 can be viewed as a kind of such a visualization method. A number of visualization methods have been studied in the field of MCDM [33].

The other class involves dimensionality reduction. In Obayashi and Sakai [30], self-organizing maps (SOM) were used to visualize obtained solutions of a four-objective supersonic wing design problem. Yoshikawa and Furuhashi [31] examined the use of a number of multivariate data analysis methods such as SOM and ICA (independent component analysis) for the visualization of six thousand solutions of a seven-objective nurse scheduling problem. In these studies, visualization is handled as a data mining task.

On the other hand, Köppen and K. Yoshida [32] used the preservation of the Pareto dominance relation as a criterion of dimensionality reduction for visualization. This is the same idea as [28], [29] in the previous subsection. One interesting idea in [32] is a multiobjective formulation of dimensionality reduction with two criteria: the number of objectives and the preservation of the Pareto dominance relation.

H. Many-Objective Test Problems

For many-objective optimization, seven test problems were proposed by Deb et al. [54], which are called DTLZ test problems (DTLZ1 to DTLZ7). The main feature of the DTLZ test problems is its scalability: the number of objectives can be arbitrarily specified. Two problems were added to the seven DTLZ test problems by Deb et al. [55]. Some test problems have different indexes between the two papers (i.e., DTLZ5 [54] => DTLZ6 [55], 6 => 7, and 7 => 8).

Whereas the DTLZ test problems have often been used in the literature, some combinatorial problems such as knapsack problems and traveling salesman problems (TSP) have also been used as many-objective test problems. In Table I, we summarize test problems used in the above-mentioned studies on evolutionary many-objective optimization.

TABLE I
TEST PROBLEMS USED IN STUDIES ON MANY-OBJECTIVE OPTIMIZATION

References	Problems	Objectives
[12]	Knapsack	2, 3, 4, 5
[13]	Heuristic learning	7, 8
[14]	Nurse scheduling	25
[15]	DTLZ	2, 8, 15
[16]	TSP, Job shop scheduling	5, 10, 15, 20
[17]	DTLZ	2-10, 15, 20, 25, ..., 50
[18]	DTLZ	3, 4, 5, 6
[19]-[21]	Knapsack	2, 3, 4
[22]	Tanaka et al. (1995)	2, 5
[23]	Flight control system	8
[24]	DTLZ	2, 5, 10
[25]	Miettinen et al. (2003)	5
[26]	Modified DTLZ	5, 10, 20, 30
[27]	Modified DTLZ	3, 5, 10, 20, 30, 50
[28]	Knapsack, DTLZ	5, 15, 25
[29]	DTLZ, Modified DTLZ	5, 7, 9
[30]	Supersonic wing design	4
[31]	Nurse scheduling	7
[32]	DTLZ	15

IV. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this paper, we first explained some difficulties of EMO algorithms for many-objective optimization by computational experiments using NSGA-II. Then we reviewed the area of evolutionary many-objective optimization. Some approaches in this area try to find a set of non-dominated solutions that approximates the entire Pareto front. For example, good results were obtained by IBEAs in [18]. One important future research issue is the decrease in the computation cost in hypervolume calculation. The use of alternative indicators for many-objective optimization (instead of hypervolume) seems to be a promising research issue as well as the computation cost reduction in hypervolume calculation. A set of uniform scalarizing functions can be used as an alternative indicator since their calculation is easy even for many objectives. Other approaches focus on a specific region in a high-dimensional objective space using not only Pareto dominance but also decision maker's preference. It seems that more sophisticated methods can be devised by the collaboration between the EMO community and the MCDM community.

REFERENCES

[1] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, Chichester, 2001.

[2] C. A. C. Coello, D. A. van Veldhuizen, and G. B. Lamont, *Evolutionary Algorithms for Solving Multi-Objective Problems*, Kluwer, Boston, 2002.

[3] C. A. C. Coello and G. B. Lamont, *Applications of Multi-Objective Evolutionary Algorithms*, World Scientific, Singapore, 2004.

[4] A. Abraham, L. C. Jain, and R. Goldberg (eds.), *Evolutionary Multiobjective Optimization: Theoretical Advances and Applications*, Springer, Berlin, 2005.

[5] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach," *IEEE Trans. on Evolutionary Computation* 3 (4), pp. 257-271, November 1999.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation* 6 (2), pp. 182-197, April 2002.

[7] V. Khara, X. Yao, and K. Deb, "Performance scaling of multi-objective evolutionary algorithms," *Lecture Notes in Computer Science 2632: Evolutionary Multi-Criterion Optimization - EMO 2003*, pp. 367-390, Springer, Berlin, April 2003.

[8] R. C. Purshouse and P. J. Fleming, "Evolutionary many-objective optimization: An exploratory analysis," *Proc. of 2003 IEEE Congress on Evolutionary Computation*, pp. 2066-2073, Canberra, December 8-12, 2003.

[9] A. Jaszkiewicz, "On the computational efficiency of multiple objective metaheuristics: The knapsack problem case study," *European Journal of Operational Research* 158 (2), pp. 418-433, October 2004.

[10] E. J. Hughes, "Evolutionary many-objective optimization: Many once or one many?," *Proc. of 2005 IEEE Congress on Evolutionary Computation*, pp. 222-227, Edinburgh, September 2-5, 2005.

[11] H. Ishibuchi, Y. Nojima, and T. Doi, "Comparison between single-objective and multi-objective genetic algorithms: Performance comparison and performance measures," *Proc. of 2006 IEEE Congress on Evolutionary Computation*, pp. 3959-3966, Vancouver, July 16-21, 2006.

[12] H. Sato, H. E. Aguirre, and K. Tanaka, "Controlling dominance area of solutions and its impact on the performance of MOEAs," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 5-20, Springer, Berlin, March 2007.

[13] N. Drechsler, R. Drechsler, and B. Becker, "Multi-objective optimization based on relation favour," *Lecture Notes in Computer Science 1993: Evolutionary Multi-Criterion Optimization - EMO 2001*, pp. 154-166, Springer, Berlin, March 2001.

[14] A. Sülflow, N. Drechsler, and R. Drechsler, "Robust multi-objective optimization in high dimensional spaces," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 715-726, Springer, Berlin, March 2007.

[15] M. Köppen and K. Yoshida, "Substitute distance assignments in NSGA-II for handling many-objective optimization problems," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 727-741, Springer, Berlin, March 2007.

[16] D. Corne and J. Knowles, "Techniques for highly multiobjective optimization: Some non-dominated points are better than others," *Proc. of 2007 Genetic and Evolutionary Computation Conference*, pp. 773-780, London, July 7-11, 2007.

[17] S. Kukkonen and J. Lampinen, "Ranking-dominance and many-

objective optimization," *Proc. of 2007 IEEE Congress on Evolutionary Computation*, pp. 3983-3990, Singapore, September 25-28, 2007.

[18] T. Wagner, N. Beume, and B. Naujoks, "Pareto-, aggregation-, and indicator-based methods in many-objective optimization," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 742-756, Springer, Berlin, March 2007.

[19] H. Ishibuchi, N. Tsukamoto, and Y. Nojima, "Iterative approach to indicator-based multiobjective optimization," *Proc. of 2007 IEEE Congress on Evolutionary Computation*, pp. 3697-3704, Singapore, September 25-28, 2007.

[20] H. Ishibuchi, T. Doi, and Y. Nojima, "Incorporation of scalarizing fitness functions into evolutionary multiobjective optimization algorithms," *Lecture Notes in Computer Science 4193: Parallel Problem Solving from Nature - PPSN IX*, pp. 493-502, Springer, Berlin, September 2006.

[21] H. Ishibuchi and Y. Nojima, "Optimization of scalarizing functions through evolutionary multiobjective optimization," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 51-65, Springer, Berlin, March 2007.

[22] E. J. Hughes, "MSOPS-II: A general-purpose many-objective optimiser," *Proc. of 2007 IEEE Congress on Evolutionary Computation*, pp. 3944-3951, Singapore, September 25-28, 2007.

[23] P. J. Fleming, R. C. Purshouse, and R. J. Lygoe, "Many-objective optimization: An engineering design perspective," *Lecture Notes in Computer Science 3410: Evolutionary Multi-Criterion Optimization - EMO 2005*, pp. 14-32, Springer, Berlin, March 2005.

[24] K. Deb and J. Sundar, "Preference point based multi-objective optimization using evolutionary algorithms," *Proc. of 2006 Genetic and Evolutionary Computation Conference*, pp. 635-642, Seattle, July 8-12, 2006.

[25] L. Thiele, K. Miettinen, P. J. Korhonen, and J. Molina, "A preference-based interactive evolutionary algorithm for multiobjective optimization," *Helsinki School of Economics, Working Paper*, W-412, January 2007.

[26] K. Deb and D. K. Saxena, "On finding Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems," *KanGAL Report*, No. 2005011, Kanpur Genetic Algorithms Laboratory (KanGAL), Indian Institute of Technology Kanpur, 2005.

[27] K. Deb and K. Saxena, "Searching for Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems," *Proc. of 2006 IEEE Congress on Evolutionary Computation*, pp. 3353-3360, Vancouver, July 16-21, 2006.

[28] D. Brockhoff and E. Zitzler, "Are all objectives necessary? On dimensionality reduction in evolutionary multiobjective optimization," *Lecture Notes in Computer Science 4193: Parallel Problem Solving from Nature - PPSN IX*, pp. 533-542, Springer, Berlin, September 2006.

[29] D. Brockhoff and E. Zitzler, "Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods," *Proc. of 2007 IEEE Congress on Evolutionary Computation*, pp.2086-2093, Singapore, September 25-28, 2007.

[30] S. Obayashi and D. Sasaki, "Visualization and data mining of Pareto solutions using self-organizing map," *Lecture Notes in Computer Science 2632: Evolutionary Multi-Criterion Optimization - EMO 2003*, pp. 796-809, Springer, Berlin, April 2003.

[31] T. Yoshikawa and T. Furuhashi, "Visualization techniques for mining of solutions," *Proc. of 8th International Symposium on Advanced Intelligent Systems*, pp. 68-71, Sokcho, Korea, September 5-8, 2007.

[32] M. Köppen and K. Yoshida, "Visualization of Pareto-sets in evolutionary multi-objective optimization," *Proc. of 7th International Conference on Hybrid Intelligent Systems*, Kaiserslautern, Germany, pp. 156-161, September 17-19, 2007.

[33] K. Miettinen, *Nonlinear Multiobjective Optimization*, Kluwer, Boston, 1998.

[34] A. Jaszkiewicz, "On the performance of multiple-objective genetic local search on the 0/1 knapsack problem - A comparative experiment," *IEEE Trans. on Evolutionary Computation* 6 (4), pp. 402-412, August 2002.

[35] K. Ikeda, H. Kita, and S. Kobayashi, "Failure of Pareto-based MOEAs: Does non-dominated really mean near to optimal?" *Proc. of 2001 IEEE Congress on Evolutionary Computation*, pp. 957-962, Seoul, May 27-30, 2001.

[36] J. Branke, T. Kaußler, and H. Schmeck, "Guidance in evolutionary multi-objective optimization," *Advances in Engineering Software* 32 (6), pp. 499-507, 2001.

[37] J. Branke and K. Deb, "Integrating user preferences into evolutionary multi-objective optimization," in Y. Jin (ed.), *Knowledge Incorporation in Evolutionary Computation*, pp. 461-477, Springer, Heidelberg, 2005.

[38] J. D. Knowles and D. W. Corne, "On metrics for comparing non-dominated sets," *Proc. of 2002 IEEE Congress on Evolutionary Computation*, pp. 711-716, Honolulu, May 12-17, 2002.

[39] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: An analysis and review," *IEEE Trans. on Evolutionary Computation*. (7) 2, pp. 117-132, April 2003.

[40] T. Okabe, Y. Jin, and B. Sendhoff, "A critical survey of performance indices for multi-objective optimization," *Proc. of 2003 IEEE Congress on Evolutionary Computation*, pp. 878-885, Canberra, Australia, December 8-12, 2003.

[41] E. Zitzler and L. Thiele, "Multiobjective optimization using evolutionary algorithms – A comparative case study," *Lecture Notes in Computer Science 1498: Parallel Problem Solving from Nature - PPSN V*, pp. 292-301, Springer, Berlin, September 1998.

[42] J. D. Knowles, D. W. Corne, and M. Fleischer, "Bounded archiving using the Lebesgue measure," *Proc. of 2003 Congress on Evolutionary Computation*, pp. 2490-2497, Canberra, December 8-12, 2003.

[43] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," *Lecture Notes in Computer Science 3242: Parallel Problem Solving from Nature - PPSN VIII*, pp. 832-842, Springer, Berlin, 2004.

[44] M. Emmerich, N. Beume, and B. Naujoks, "An EMO algorithm using the hypervolume measure as selection criterion," *Lecture Notes in Computer Science 3410: Evolutionary Multi-Criterion Optimization - EMO 2005*, pp. 62-76, Springer, Berlin, 2005.

[45] N. Beume, B. Naujoks, and M. Emmerich, "SMS-EMOA: multiobjective selection based on dominated hypervolume," *European Journal of Operational Research* 180 (3), pp. 1653-1669, September 2007.

[46] E. Zitzler, D. Brockhoff, and L. Thiele, "The hypervolume indicator revisited: On the design of Pareto-compliant indicators via weighted integration," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 862-876, Springer, Berlin, 2007.

[47] E. J. Hughes, "Multiple single objective Pareto sampling," *Proc. of 2003 IEEE Congress on Evolutionary Computation*, pp. 2678-2684, Canberra, December 8-12, 2003.

[48] H. Ishibuchi and T. Murata, "A multi-objective genetic local search algorithm and its application to flowshop scheduling," *IEEE Trans. on Systems, Man, and Cybernetics - Part C* 28 (3), pp. 392-403, August 1998.

[49] A. Jaszkiewicz, "Genetic local search for multi-objective combinatorial optimization," *European Journal of Operational Research* 137 (1), pp. 50-71, February 2002.

[50] H. Ishibuchi, T. Yoshida, and T. Murata, "Balance between genetic search and local search in memetic algorithms for multiobjective permutation flowshop scheduling," *IEEE Trans. on Evolutionary Computation* 7 (2), pp. 204-223, April 2003.

[51] C. M. Fonseca and P. J. Fleming, "Multiobjective optimization and multiple constraint handling with evolutionary algorithms - Part I: A unified formulation," *IEEE Trans. on Systems, Man and Cybernetics - Part A* 28 (1), pp. 38-47, January 1998.

[52] C. A C. Coello, "Handling preferences in evolutionary multiobjective optimization: A survey," *Proc. of 2000 IEEE Congress on Evolutionary Computation*, pp. 30-37, San Diego, July 16-19, 2000.

[53] D. Cvetkovic and P. Parmee, "Preferences and their application in evolutionary multiobjective optimization," *IEEE Trans. on Evolutionary Computation* 6 (1), pp. 42-57, February 2002.

[54] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multi-objective optimization test problems," *Proc. of 2002 IEEE Congress on Evolutionary Computation*, pp.825-830, Honolulu, May 12-17, 2002.

[55] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable test problems for evolutionary multiobjective optimization," in A. Abraham, L.C. Jain, and R. Goldberg (eds.), *Evolutionary Multiobjective Optimization: Theoretical Advances and Applications*, pp. 105-145, Springer, Berlin, 2005.