An Empirical Study on the Effect of Mating Restriction on the Search Ability of EMO Algorithms

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Abstract. This paper examines the effect of mating restriction on the search ability of EMO algorithms. First we propose a simple but flexible mating restriction scheme where a pair of similar (or dissimilar) individuals is selected as parents. In the proposed scheme, one parent is selected from the current population by the standard binary tournament selection. Candidates for a mate of the selected parent are winners of multiple standard binary tournaments. The selection of the mate among multiple candidates is based on the similarity (or dissimilarity) to the first parent. The strength of mating restriction is controlled by the number of candidates (i.e., the number of tournaments used for choosing candidates from the current population). Next we examine the effect of mating restriction on the search ability of EMO algorithms to find all Pareto-optimal solutions through computational experiments on small test problems using the SPEA and the NSGA-II. It is shown that the choice of dissimilar parents improves the search ability of the NSGA-II on small test problems. Then we further examine the effect of mating restriction using large test problems. It is shown that the choice of similar parents improves the search ability of the SPEA and the NSGA-II to efficiently find near Pareto-optimal solutions of large test problems. Empirical results reported in this paper suggest that the proposed mating restriction scheme can improve the performance of EMO algorithms for many test problems while its effect is problem-dependent and algorithm-dependent.

1 Introduction

Since Schaffer's study [13], evolutionary algorithms have been applied to various multiobjective optimization problems for finding their Pareto-optimal solutions (e.g., see Coello et al. [1] and Deb [3]). Those algorithms are often referred to as EMO (evolutionary multiobjective optimization) algorithms. Recent EMO algorithms usually share some common ideas such as elitism, fitness sharing and Pareto ranking. While mating restriction has been often discussed in the literature, it has not been used in many EMO algorithms as pointed out in some reviews on EMO algorithms [6, 14, 18]. The aim of this paper is to examine the effect of mating restriction on the search ability of EMO algorithms. More specifically, we demonstrate how the search ability of EMO algorithms to find Pareto-optimal or near Pareto-optimal solutions can be improved by mating restriction.

Mating restriction was suggested by Goldberg [7] for single-objective genetic algorithms. Hajela & Lin [8] and Fonseca & Fleming [5] used it in their EMO algorithms. The basic idea of mating restriction is to ban the crossover of dissimilar parents from which good offspring are not likely to be generated. In the implementation of mating restriction, a user-definable parameter σ_{mating} called the mating radius is usually used for banning the crossover of two parents whose distance is larger than σ_{mating} . The distance between two parents is measured in the decision space or the objective space. The necessity of mating restriction in EMO algorithms was also stressed by Jaszkiewicz [11] and Watanabe et al. [15]. On the other hand, Zitzler & Thiele [17] reported that no improvement was achieved by mating restriction in their computational experiments. Van Veldhuizen & Lamont [14] mentioned that the empirical evidence presented in the literature could be interpreted as an argument either for or against the use of mating restriction. Moreover, there was also an argument for the selection of dissimilar parents. Horn et al. [9] argued that information from very different types of tradeoffs could be combined to yield other kinds of good tradeoffs. Schaffer [13] examined the selection of dissimilar parents but observed no improvement.

In this paper, we examine the effect of mating restriction on the search ability of EMO algorithms through computational experiments on multiobjective knapsack and permutation flowshop scheduling problems. As EMO algorithms, we use the SPEA [18] and the NSGA-II [4] because their high search ability was empirically demonstrated in the literature [3, 4, 16, 18]. We first propose a simple but flexible mating restriction scheme for implementing the selection of dissimilar parents as well as similar parents in a unified framework. Next we examine the effect of mating restriction on the search ability of the SPEA and the NSGA-II to find all Pareto-optimal solutions of small test problems. Then we examine their search ability to efficiently find near Pareto-optimal solutions of large test problems. Experimental results clearly show that the search ability of those EMO algorithms on some test problems can be improved by mating restriction. It is also shown that the effect of mating restriction is problem-dependent and algorithm-dependent.

2 Mating Restriction Scheme without Mating Radius

In general, an n-objective optimization problem can be written as

$$\text{Optimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}), \ldots,\ f_n(\mathbf{x}))\,, \tag{1}$$

$$\text{subject to } \mathbf{x} \in \mathbf{X}\,, \tag{2}$$

where $\mathbf{f}(\mathbf{x})$ is the objective vector, $f_i(\mathbf{x})$ is the i-th objective to be minimized or maximized, $\mathbf{x}$ is the decision vector, and $\mathbf{X}$ is the feasible region in the decision space.

Let us denote the distance between two solutions $\mathbf{x}$ and $\mathbf{y}$ as $|\mathbf{x}-\mathbf{y}|$ in the decision space and $|\mathbf{f}(\mathbf{x})-\mathbf{f}(\mathbf{y})|$ in the objective space. In this paper, the distance $|\mathbf{f}(\mathbf{x})-\mathbf{f}(\mathbf{y})|$ in the objective space is measured by the Euclidean distance as

$$|\mathbf{f}(\mathbf{x})-\mathbf{f}(\mathbf{y})| = \sqrt{|f_1(\mathbf{x})-f_1(\mathbf{y})|^2 + \cdots + |f_n(\mathbf{x})-f_n(\mathbf{y})|^2}\,. \tag{3}$$

On the other hand, the definition of the distance $|\mathbf{x}-\mathbf{y}|$ in the decision space totally depends on the representation of solutions in a particular problem. For example, we use the Hamming distance for m-item knapsack problems as

$$|\mathbf{x}-\mathbf{y}| = |x_1-y_1| + \cdots + |x_m-y_m|, \tag{4}$$

where $\mathbf{x}$ and $\mathbf{y}$ are binary strings of the length m: $\mathbf{x}=x_1x_2\cdots x_m$ and $\mathbf{y}=y_1y_2\cdots y_m$. On the other hand, solutions of permutation flowshop scheduling problems are permutations of given jobs. In this case, we use the sum of the distance between the positions of each job as the distance of two solutions. The calculation of the distance is illustrated in Fig. 1. The distance between the positions of Job 1 (denoted by J1 in Fig. 1) is 4 since it is placed in the first position of String 1 and the fifth position of String 2. The distance between the positions of the other jobs is calculated in the same manner (i.e., 1 for Job 2, 0 for Job 3 and Job 4, and 3 for Job 5). Thus the distance between the two strings in Fig. 1 is calculated as 8 (i.e., $4+1+0+0+3$).

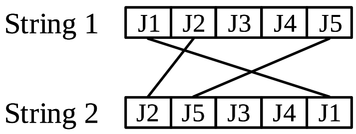


Fig. 1. Distance between two strings for five-job flowshop scheduling problems.

In this paper, we propose a mating restriction scheme for examining the effect of mating restriction on the search ability of EMO algorithms. The proposed mating restriction scheme is illustrated in Fig. 2 where open circles at the bottom denote individuals randomly drawn from the current population with replacement. One parent (i.e., Parent A in Fig. 2) is chosen by the standard binary tournament selection. In the selection of a mate for Parent A (i.e., Parent B in Fig. 2), first the standard binary tournament selection is iterated β times for finding β candidates. Each candidate is the winner of a tournament. Then a mate is chosen among the β candidates by measuring the distance from each candidate to Parent A. The distance is measured in the decision or objective space. The most similar (or dissimilar) candidate with the minimum (or maximum) distance to Parent A is selected as its mate. In this manner, a mate for Parent A is selected through two-stage tournament selection. In the first stage, the fitness-based binary tournament selection is iterated for finding β candidates. In the second stage, the distance-based tournament selection of the tournament size β is performed for choosing a single individual as a mate for Parent A from the β winners in the first stage. Our mating restriction scheme has the following flexibility in its implementation:

(a) The choice between the decision space and the objective space in which the distance is measured.

(b) The choice between the similarity (i.e., minimum distance) and the dissimilarity (i.e., maximum distance) as the mate selection criterion in the distance-based tournament selection in the second stage.

(c) The value of β, i.e., the number of candidates from which a mate is chosen based on the mate selection criterion.

The user-definable parameter β can be viewed as the strength of mating restriction. That is, the strength of mating restriction is controllable through the value of β. When $\beta = 1$, our mating restriction scheme is the same as the standard binary tournament selection with no mating restriction. As the value of β increases, more similar (or dissimilar) parents are selected and recombined.

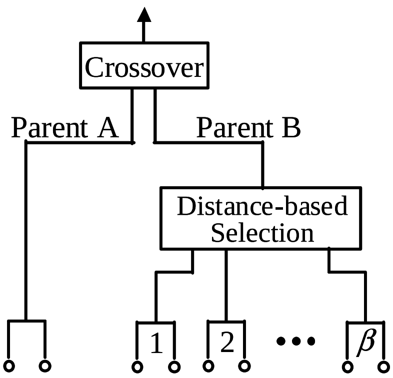


Fig. 2. Our mating restriction scheme.

3 Examination of the Effect of Mating Restriction

In this section, we examine the effect of the proposed mating restriction scheme on the performance of EMO algorithms through computational experiments on multiobjective knapsack and permutation flowshop scheduling problems. For this purpose, we combined our mating restriction scheme with recently developed popular EMO algorithms: the SPEA [18] and the NSGA-II [4]. As mentioned in the previous section, our mating restriction scheme is the same as the standard binary tournament selection when $\beta = 1$. Thus the modified SPEA and the modified NSGA-II are the same as their original versions when $\beta = 1$.

3.1 Test Problems

In our computational experiments, we used four knapsack problems in Zitzler & Thiele [18]: two-objective 250-item, three-objective 250-item, two-objective 500-item, and three-objective 500-item test problems. We also generated 10 small test problems with two objectives and 30 items in the same manner as [18]. The small test problems were used for examining the search ability of the EMO algorithms to find all Pareto-optimal solutions while the large test problems were used for examining their search ability to efficiently find near Pareto-optimal solutions. We also generated 10 two-objective permutation flowshop scheduling problems with 10 machines and 12 jobs in the same manner as Ishibuchi & Murata [10]. The two objectives are the minimization of the makespan and the maximum tardiness. As large permutation flowshop scheduling problems, we generated four test problems with 20 machines: two-objective 40-job, three-objective 40-job, two-objective 80-job, and three-objective 80-job problems. In the three-objective problems, the minimization of the total flow time was used in addition to the minimization of the makespan and the maximum tardiness.

3.2 Parameter Specifications

We examined all the four combinations related to the distance definition and the mate selection criterion in our mating restriction scheme: {decision space, objective space} $\times$ {minimum distance, maximum distance}. We also examined ten different values of β : $\beta =$ 1,2,3,...,10. The SPEA and the NSGA-II combined with our mating restriction scheme were applied to knapsack problems with m items under the following parameter specifications:

Crossover probability: 0.8,
Mutation probability: $1/m$,
Population size in NSGA-II: 200,
Population size in SPEA: 100,
Population size of the secondary population in SPEA: 100,
Stopping condition: 2000 generations.

The above specifications of the population size seem to somewhat favor the NSGA-II because 200 solutions were examined in each generation of the NSGA-II while 100 solutions were examined in the SPEA. This is, however, not a serious problem because our aim in this paper is to examine the effect of mating restriction on the search ability of each algorithm (not to compare them with each other).

We also used the same parameter specifications for flowshop scheduling except for the mutation probability. The mutation probability was defined for each string as 0.5 (for details of genetic operations for flowshop scheduling, see [10]). It should be noted that the mutation was applied to each string in flowshop scheduling while it was applied to each bit in knapsack problems.

3.3 Performance Measures

Various performance measures have been proposed in the literature for evaluating a set of non-dominated solutions. As explained in Knowles & Corne [12], no single performance measure can simultaneously evaluate various aspects of a solution set. Moreover, some performance measures are not designed for simultaneously comparing many solution sets but for comparing two solution sets with each other.

For the small test problems (i.e., 30-item knapsack problems and 12-job flowshop scheduling problems), we used the ratio of undiscovered Pareto-optimal solutions as a performance measure. This ratio is referred to as the undiscovered solution ratio in this paper. For calculating this ratio, all Pareto-optimal solutions of the small test problems were found by an enumeration method. The average number of the Pareto-optimal solutions was 17.4 in the knapsack problems and 13.6 in the flowshop scheduling problems. For the large test problems, we used the average distance from each Pareto-optimal solution to its nearest solution in a solution set as a performance measure. This measure was used in Czyzak & Jaszkiewicz [2] and referred to as $\mathrm{D1_R}$ in Knowles & Corne [12]. For any multiobjective optimization problem, it is reasonable for the decision maker (DM) to choose a final solution $\mathbf{x}^*$ from the Pareto-optimal solution set. The final solution $\mathbf{x}^*$ is the best solution with respect to the DM's preference. When the true Pareto-optimal solution set is not given, the DM will choose a final solution $\mathbf{x}$ from an available solution set. When the available solution set is a good approximation of the true Pareto-optimal solution set, the chosen solution $\mathbf{x}$ may be close to $\mathbf{x}^*$. In this case, the loss due to choosing $\mathbf{x}$ instead of $\mathbf{x}^*$ can be approximately measured by the distance between $\mathbf{x}$ and $\mathbf{x}^*$ in the objective space. Since $\mathbf{x}$ and $\mathbf{x}^*$ are unknown, we cannot directly measure the distance. The expected value of the distance, however, can be roughly estimated by the average value of the distance from each Pareto-optimal solution to its nearest available solution. The $\mathrm{D1_R}$ measure corresponds to this approximation.

The $\mathrm{D1_R}$ measure needs all Pareto-optimal solutions of each test problem. Since all Pareto-optimal solutions of the two-objective 250-item and 500-item knapsack problems were available from the homepage of the first author of [18], we used them. For the three-objective 250-item and 500-item knapsack problems, we found near Pareto-optimal solutions using the SPEA and the NSGA-II. These algorithms were applied to each test problem using longer CPU time and larger memory storage (i.e., 30000 generations with the population size 200 and the secondary population of the same size in the SPEA, and 30000 generations with the population size 400 in the NSGA-II) than the other computational experiments (see Subsection 3.2). We also used a single-objective genetic algorithm with a secondary population where all the non-dominated solutions were stored with no size limitation. Each of the three objectives was used in the single-objective genetic algorithm. This algorithm was applied to each three-objective test problem 30 times (10 times for each objective using the same stopping condition as the NSGA-II: 30000 generations with the population size 400). The SPEA and the NSGA-II were also applied to each test problem 10 times. Thus we obtained 50 solution sets for each test problem. Then we chose non-dominated solutions from the obtained 50 solution sets as near Pareto-optimal solutions. For 40-job and 80-job flowshop scheduling problems, near Pareto-optimal solutions were found in the same manner (the stopping condition was specified as 50000 generations). The number of Pareto-optimal or near Pareto-optimal solutions for each test problem is summarized in Table 1. While each objective of the knapsack problems has the same order of magnitude, they are not the same in the flowshop scheduling problems. Thus the objective space of each flowshop scheduling problem was normalized using the obtained near Pareto-optimal solutions when the $\mathrm{D1_R}$ measure was calculated. More specifically, the objective space was normalized so that the minimum and maximum values of each objective among the near Pareto-optimal solutions became 0 and 100, respectively.

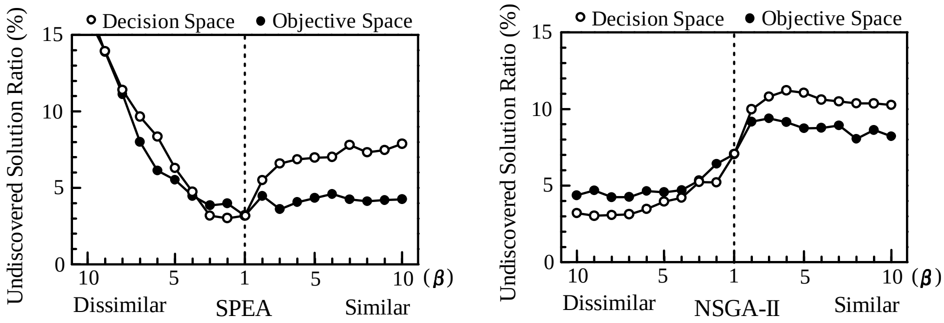
As a performance measure of a solution set (say S_j), we also used the ratio of non-dominated solutions $|S_j^*|/|S_j|$ where S_j^* is a set of solutions in S_j that are not dominated by any other solutions in other solution sets when multiple solution sets are compared with each other.

Table 1. The number of Pareto-optimal or near Pareto-optimal solutions.

Knapsack problems				Flowshop scheduling problems			
Two-objective		Three-objective		Two-objective		Three-objective	
250-item	500-item	250-item	500-item	40-job	80-job	40-job	80-job
567	1427	2158	2142	98	87	973	974

3.4 Results on Small Test Problems

In Fig. 3 (a), we show the average ratio of undiscovered Pareto-optimal solutions by the SPEA with our mating restriction scheme. The average ratio was calculated over 50 runs of the SPEA with each specification of β on each of the 10 small knapsack problems (i.e., over 500 solution sets in total for each value of β). The horizontal axis of this figure is the value of β. As shown in this figure, $\beta = 1$ corresponds to the original SPEA. The maximum distance was used in the left half of this figure as the mate selection criterion while the minimum distance was used in the right half. That is, the most dissimilar solution was selected among β candidates in the left half while the most similar solution was selected in the right half. Open circles and closed circles show the results when the distance was measured in the decision space and the objective space, respectively. In this figure, we cannot observe any significant improvement by mating restriction (by specifying β as $\beta > 1$).



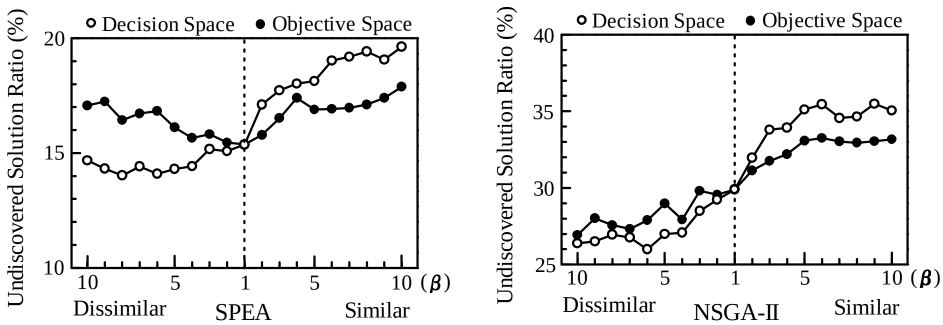
(a) Results by SPEA. (b) Results by NSGA-II.

Fig. 3. Results on the two-objective 30-item knapsack problems.

Fig. 3 (b) shows the average ratio of undiscovered Pareto-optimal solutions by the NSGA-II with our mating restriction scheme. While the search ability of the SPEA was not improved by our mating restriction scheme in Fig. 3 (a), we can observe clear improvement in the search ability of the NSGA-II in Fig. 3 (b). Large improvement was achieved in a wide range of β in the left half of Fig. 3 (b) independent of the choice between the decision space and the objective space (while somewhat better results were obtained from the distance in the decision space than the objective space). When the distance was calculated in the decision space, the average ratio of undiscovered solutions was improved from 7.05% in the case of $\beta = 1$ (i.e., the original NSGA-II) to about 3% by our mating restriction scheme (see open circles in the left half of Fig. 3 (b)). The improvement is statistically significant with the 99% confidence level for the results by $\beta \geq 2$ in the decision space and $\beta \geq 3$ in the objective space (the Mann-Whitney U test).

The average ratio of undiscovered Pareto-optimal solutions was also calculated over 50 runs of the SPEA and the NSGA-II for each of the ten small flowshop scheduling problems. Results by the SPEA and the NSGA-II were shown in Fig. 4 (a) and Fig. 4 (b), respectively. In Fig. 4 (a), the search ability of the SPEA was improved by our mating restriction scheme when the distance was measured in the decision space and dissimilar parents were chosen (i.e., open circles in the left half). The improvement is statistically significant with the 95% confidence level for the results by $\beta = 5$, 6, 7, 8, 9. On the other hand, the search ability of the NSGA-II was improved by our mating restriction scheme when the distance was measured in the objective space as well as in the decision space in the left half of Fig. 4 (b). The improvement is statistically significant with the 99% confidence level for the results by $\beta \geq 4$ in the decision space and $\beta = 4$, 6, 7, 8, 9, 10 in the objective space.

The experimental results in this subsection show that the search ability of the NSGA-II to find all Pareto-optimal solutions of the small test problems was improved by choosing dissimilar parents using our mating restriction scheme. On the other hand, the search ability of the SPEA was not improved by our mating restriction scheme except for the case of the 12-job flowshop scheduling problems where the distance was measured in the decision space (i.e., open circles in the left half of Fig. 4 (a)).

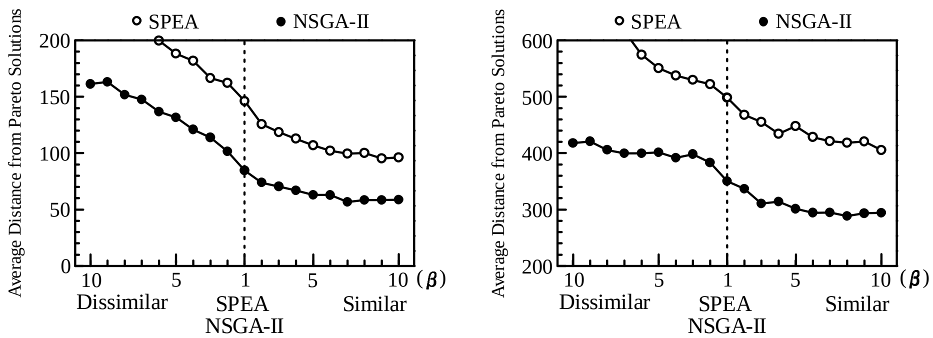


(a) Results by SPEA. (b) Results by NSGA-II.

Fig. 4. Results on the two-objective 12-job permutation flowshop scheduling problems.

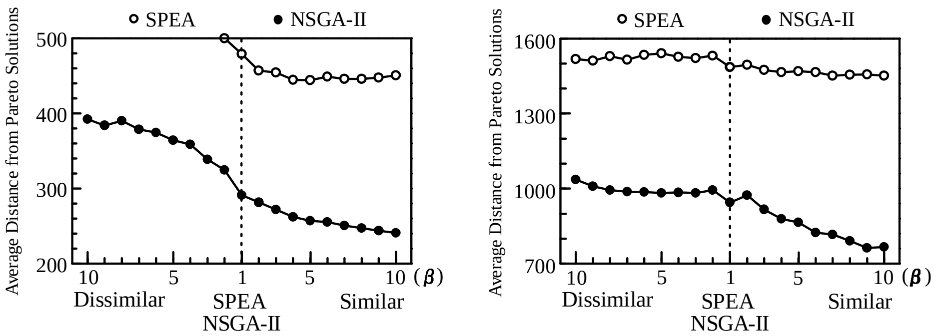
3.5 Results on Large Test Problems

We examined the search ability of the SPEA and the NSGA-II to efficiently find near Pareto-optimal solutions through computational experiments on larger knapsack problems. Each algorithm was applied to each test problem 50 times using each specification of β. Due to the page limitation, we only report experimental results for the case where the distance was measured in the objective space. In Fig. 5 and Fig. 6, we show experimental results for the two-objective problems and the three-objective problems, respectively. Similar results to those figures were also obtained when the distance was measured in the decision space.



(a) Results on the 250-item problem. (b) Results on the 500-item problem.

Fig. 5. Results on the two-objective knapsack problems.



(a) Results on the 250-item problem. (b) Results on the 500-item problem.

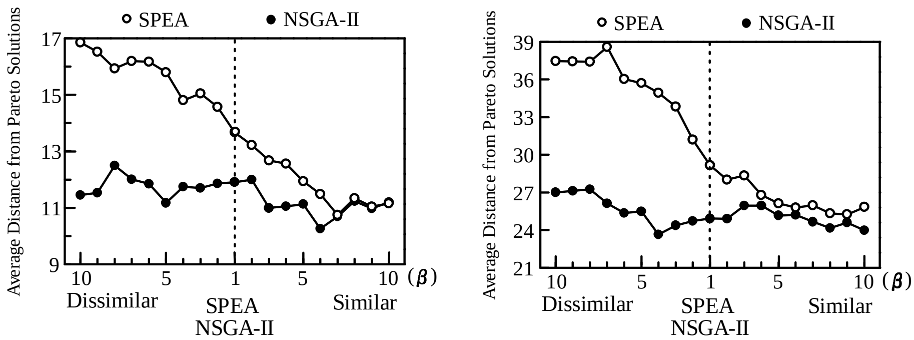
Fig. 6. Results on the three-objective knapsack problems.

In Fig. 5 and Fig. 6, we can observe the improvement of the search ability of the SPEA and the NSGA-II by choosing similar parents. We examined the statistical significance of the improvement from the case of $\beta = 1$ (i.e., the original SPEA and NSGA-II) to the case of $\beta = 10$ (i.e., the right-most open and closed circles in each figure) for three confidence levels: 95%, 97.5% and 99%. Confidence levels of the improvement are summarized in Table 2 where each knapsack problem is denoted by the number of objectives and the number of items. From this table, we can conclude that the performance of the SPEA and the NSGA-II for the 250-item and 500-item knapsack problems was significantly improved by choosing similar parents.

Table 2. Confidence level of the improvement of each algorithm for each knapsack problem by choosing similar parents using our mating restriction scheme with $\beta = 10$.

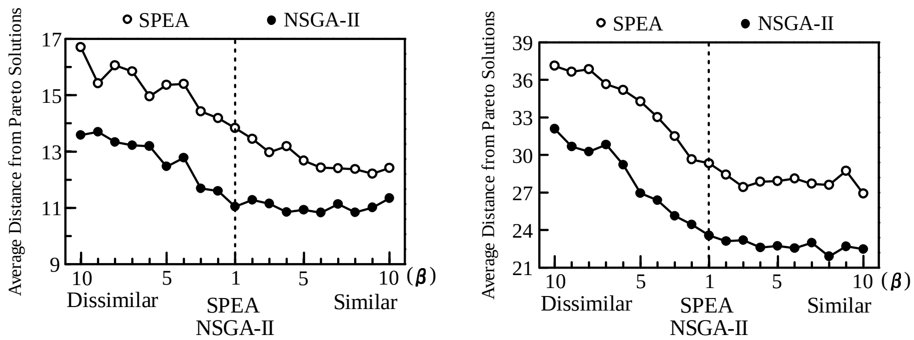
Problem	SPEA	NSGA-II
2/250	99	99
2/500	99	99
3/250	99	99
3/500	97.5	99

We show experimental results on the flowshop scheduling problems in Fig. 7 and Fig. 8 where the search ability of the SPEA was clearly improved by choosing similar parents. The improvement for the NSGA-II, however, was not clear. In the same manner as Table 2, we examined the statistical significance of the improvement from the case of $\beta = 1$ to the case of $\beta = 10$. Confidence levels of the improvement are summarized in Table 3 where each flowshop scheduling problem is denoted by the number of objectives and the number of jobs. From this table, we can conclude that the performance of the SPEA for the 40-job and 80-job flowshop scheduling problems was significantly improved by choosing similar parents. On the other hand, the performance of the NSGA-II was not significantly improved except for the case of the three-objective 80-job test problem.



(a) Results on the 40-job problem. (b) Results on the 80-job problem.

Fig. 7. Results on the two-objective permutation flowshop scheduling problems.



(a) Results on the 40-job problem. (b) Results on the 80-job problem.

Fig. 8. Results on the three-objective permutation flowshop scheduling problems.

Table 3. Confidence level of the improvement of each algorithm for each flowshop scheduling problem by choosing similar parents using our mating restriction scheme with $\beta = 10$.

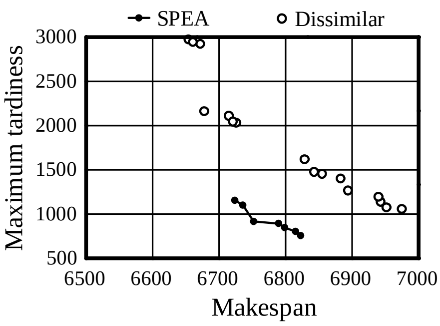
Problem	SPEA	NSGA-II
2/40	99	-
2/80	99	-
3/40	99	-
3/80	99	99

3.6 Discussions on Experimental Results

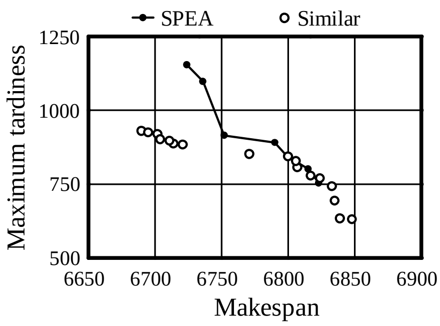
The experimental results on the small test problems in Subsection 3.4 suggest that the choice of dissimilar parents has a positive effect on the search ability of EMO algorithms to find a variety of Pareto-optimal solutions (i.e., a positive effect on the diversity of solutions). On the other hand, the experimental results on the large test problems in Subsection 3.5 suggest that the choice of similar parents has a positive effect on the search ability of EMO algorithms to efficiently find near Pareto-optimal solutions (i.e., a positive effect on the convergence speed to the Pareto-front).

These positive effects can be more clearly shown by the application to a larger permutation flowshop scheduling problem. We applied the SPEA and the NSGA-II to a two-objective 100-job permutation flowshop scheduling problem in the same manner as Subsection 3.5. We used the three variants of each algorithm: the choice of dissimilar parents with $\beta = 10$, the original algorithm with $\beta = 1$, and the choice of similar parents with $\beta = 10$. Experimental results by the SPEA and the NSGA-II are shown in Fig. 9 and Fig. 10, respectively. Each variant was applied to the 100-job permutation flowshop scheduling problem just once. We show a single solution set obtained by a single run of each variant in those figures. It should be noted that the axes of each figure are not the same because they are adjusted to the range of solution sets depicted in each figure.

In Fig. 9 (a) and Fig. 10 (a), we can observe both positive and negative effects of the choice of dissimilar parents: the increase in the diversity of solutions and the deterioration in the convergence speed to the Pareto-front. On the other hand, we can observe the positive effect of the choice of similar parents in Fig. 9 (b): the increase in the convergence speed to the Pareto-front. Actually, many solutions obtained by the original SPEA (i.e., closed circles) are clearly dominated by solutions obtained by the modified SPEA with the choice of similar parents (i.e., open circles) in Fig. 9 (b). Such improvement is not so clear for the NSGA-II in Fig. 10 (b).

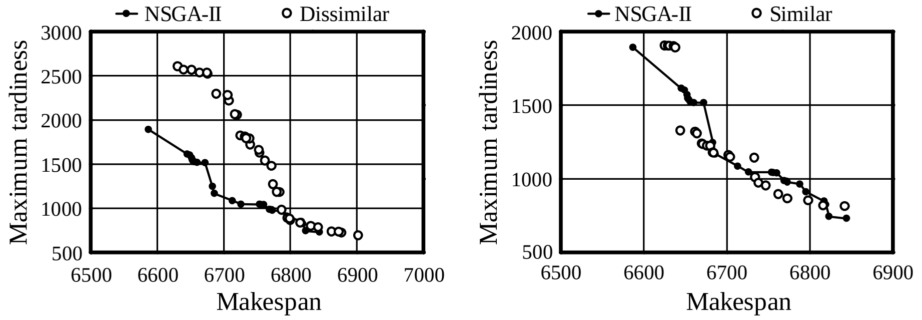


(a) Mating of dissimilar parents with $\beta = 10$.



(b) Mating of similar parents with $\beta = 10$.

Fig. 9. Solution sets obtained by the three variants of the SPEA for a 100-job problem.



(a) Mating of dissimilar parents with $\beta = 10$. (b) Mating of similar parents with $\beta = 10$.

Fig. 10. Solution sets obtained by the three variants of the NSGA-II for a 100-job problem.

The negative effect of the choice of dissimilar parents is the deterioration in the convergence speed to the Pareto-front as shown in Subsection 3.5, Fig. 9 (a) and Fig. 10 (a). On the other hand, the negative effect of the choice of similar parents is the decrease in the diversity of solutions as shown in Subsection 3.4 for the small test problems. This negative effect, however, was not clear for the large test problems.

For further examining the effect of our mating restriction scheme on the convergence speed to the Pareto-front, we calculated the average ratio of non-dominated solutions over 50 runs where five variants of each EMO algorithm were compared with each other. Experimental results are summarized in Table 4 for the SPEA and Table 5 for the NSGA-II. In those tables, K and FS mean knapsack and flowshop scheduling, respectively. Five variants in each table were compared with one another for calculating the average ratio of non-dominated solutions. From this table, we can see that the choice of similar parents increased the convergence speed while the choice of dissimilar parents decreased it.

Table 4. Average ratio of non-dominated solutions for each variant of the SPEA (%).

Problem	Dissimilar parents		SPEA	Similar parents	
	$\beta = 10$	$\beta = 5$	$\beta = 1$	$\beta = 5$	$\beta = 10$
K-2/30	89.5	96.7	99.1	97.7	97.8
K-2/250	0.2	0.0	17.5	54.7	48.2
K-2/500	0.0	0.1	25.4	61.2	31.9
K-3/250	2.4	10.2	59.4	62.4	51.8
K-3/500	1.1	4.2	67.9	67.0	36.7
FS-2/12	91.0	92.5	93.4	92.0	92.1
FS-2/40	5.4	10.9	21.1	43.4	49.6
FS-2/80	1.7	4.4	33.4	54.5	48.3
FS-3/40	4.4	9.8	36.7	52.8	54.2
FS-3/80	0.4	5.0	36.3	49.0	49.7

Table 5. Average ratio of non-dominated solutions for each variant of the NSGA-II (%).

Problem	Dissimilar parents		NSGA-II	Similar parents	
	$\beta = 10$	$\beta = 5$	$\beta = 1$	$\beta = 5$	$\beta = 10$
K-2/30	98.6	98.4	96.4	94.7	94.7
K-2/250	0.0	0.0	20.9	53.0	51.0
K-2/500	0.5	0.4	22.8	59.8	36.8
K-3/250	1.7	3.7	54.0	61.1	61.6
K-3/500	0.8	0.7	51.1	68.2	57.2
FS-2/12	87.8	86.8	87.0	84.5	84.8
FS-2/40	18.2	24.9	27.5	34.1	30.2
FS-2/80	10.9	17.3	31.7	33.1	45.2
FS-3/40	11.4	17.1	45.6	49.5	44.0
FS-3/80	2.4	6.7	41.5	47.6	50.1

4 Concluding Remarks

We examined the effect of mating restriction on the performance of the SPEA and the NSGA-II through computational experiments on multiobjective knapsack and permutation flowshop scheduling problems. Experimental results showed that the performance of these EMO algorithms on many test problems was significantly improved by mating restriction. The effect of mating restriction, however, was problem-dependent and algorithm-dependent. For example, the search ability of the NSGA-II to find all Pareto-optimal solutions of small knapsack problems was improved by choosing dissimilar parents while its search ability to efficiently find near Pareto-optimal solutions of large knapsack problems was improved by choosing similar parents. Experimental results suggest that the positive and negative effects of choosing dissimilar parents are the increase in the diversity of solutions and the deterioration in the convergence speed to the Pareto-front, respectively. Experimental results also suggest that the positive and negative effects of choosing similar parents are the increase in the convergence speed and the decrease in the diversity of solutions. If we want to improve the performance of an EMO algorithm with respect to the convergence speed to the Pareto-front, it may be worth examining the use of the proposed mating restriction scheme with the similarity as the mate selection criterion in the EMO algorithm. One of future research topics is to devise a mating restriction scheme that can improve the convergence speed to the Pareto-front without decreasing the diversity of solutions.

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References

1. Coello Coello, C. A., Van Veldhuizen, D. A., and Lamont, G. B.: *Evolutionary Algorithms for Solving Multi-Objective Problems*, Kluwer Academic Publishers, Boston (2002).
2. Czyzak, P., and Jaszkiewicz, A.: Pareto-Simulated Annealing – A Metaheuristic Technique for Multi-Objective Combinatorial Optimization, *Journal of Multi-Criteria Decision Analysis* 7 (1998) 34-47.
3. Deb, K.: *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, Chichester (2001).
4. Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II, *IEEE Trans. on Evolutionary Computation* 6 (2002) 182 –197.
5. Fonseca, C. M., and Fleming, P. J.: Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization, *Proc. of 5th International Conference on Genetic Algorithms* (1993) 416-423.
6. Fonseca, C. M., and Fleming, P. J.: An Overview of Evolutionary Algorithms in Multiobjective Optimization, *Evolutionary Computation* 3 (1995) 1-16.
7. Goldberg, D. E.: *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading (1989).
8. Hajela, P., and Lin, C. Y.: Genetic Search Strategies in Multicriterion Optimal Design, *Structural Optimization* 4 (1992) 99-107.
9. Horn, J., Nafpliotis, N., and Goldberg, D. E.: A Niched Pareto Genetic Algorithm for Multi-Objective Optimization, *Proc. of 1st IEEE International Conference on Evolutionary Computation* (1994) 82-87.
10. Ishibuchi, H., and Murata, T.: A Multi-Objective Genetic Local Search Algorithm and Its Application to Flowshop Scheduling, *IEEE Trans. on Systems, Man, and Cybernetics - Part C: Applications and Reviews* 28 (1998) 392-403.
11. Jaszkiewicz, A.: Genetic Local Search for Multi-Objective Combinatorial Optimization, *European Journal of Operational Research* 137 (2002) 50-71.
12. Knowles, J. D., and Corne, D. W.: On Metrics for Comparing Non-Dominated Sets, *Proc. of 2002 Congress on Evolutionary Computation* (2002) 711-716.
13. Schaffer, J. D.: Multiple Objective Optimization with Vector Evaluated Genetic Algorithms, *Proc. of 1st International Conference on Genetic Algorithms and Their Applications* (1985) 93-100.
14. Van Veldhuizen, D. A., and Lamont, G. B.: Multiobjective Evolutionary Algorithms: Analyzing the State-of-the-Art, *Evolutionary Computation* 8 (2000) 125-147.
15. Watanabe, S., Hiroyasu, T., and Miki, M.: LCGA: Local Cultivation Genetic Algorithm for Multi-Objective Optimization Problem, *Proc. of 2002 Genetic and Evolutionary Computation Conference* (2002) 702.
16. Zitzler, E., Deb, K., Thiele, L.: Comparison of Multiobjective Evolutionary Algorithms: Empirical Results, *Evolutionary Computation* 8 (2000) 173-195.
17. Zitzler, E., Thiele, L.: Multiobjective Optimization using Evolutionary Algorithms – A Comparative Case Study, *Proc. of 5th International Conference on Parallel Problem Solving from Nature* (1998) 292-301.
18. Zitzler, E., Thiele, L.: Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach, *IEEE Transactions on Evolutionary Computation* 3 (1999) 257-271.