H. Ishibuchi, M. Yamane, and Y. Nojima, "Difficulty in evolutionary multiobjective optimization of discrete objective functions with different granularities," Lecture Notes in Computer Science 7811: Evolutionary Multi-Criterion Optimization - EMO 2013, pp. 230-245, Springer, Berlin, March 2013.

Difficulty in Evolutionary Multiobjective Optimization of Discrete Objective Functions with Different Granularities

Hisao Ishibuchi, Masakazu Yamane and Yusuke Nojima

Department of Computer Science and Intelligent Systems, Graduate School of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531, Japan
{hisaoi@, masakazu.yamane@ci., nojima@}cs.osakafu-u.ac.jp

Abstract. Objective functions are discrete in combinatorial optimization. In general, the number of possible values of a discrete objective is totally different from problem to problem. That is, discrete objectives have totally different granularities in different problems (In this paper, "granularity" means the width of discretization intervals). In combinatorial multiobjective optimization, a single problem has multiple discrete objectives with different granularities. Some objectives may have fine granularities with many possible values while others may have very coarse granularities with only a few possible values. Handling of such a combinatorial multiobjective problem has not been actively discussed in the EMO community. In our former study, we showed that discrete objectives with coarse granularities slowed down the search by NSGA-II, SPEA2, MOEA/D and SMS-EMOA on two-objective problems. In this paper, we first discuss why such a discrete objective deteriorates the search ability of those EMO algorithms. Next we propose the use of strong Pareto dominance in NSGA-II to improve its search ability. Then we examine the effect of discrete objectives on the performance of the four EMO algorithms on many-objective problems. An interesting observation is that discrete objectives with coarse granularities improve the search ability of NSGA-II and SPEA2 on many-objective problems whereas they deteriorate their search ability on two-objective problems. The performance of MOEA/D and SMS-EMOA is always deteriorated by discrete objectives with coarse granularities. These observations are discussed from the following two viewpoints: One is the difficulty of many-objective problems for Pareto dominance-based EMO algorithms, and the other is the relation between discrete objectives and the concept of ε-dominance.

Keywords: Evolutionary multiobjective optimization, many-objective problems, discrete objectives, ε-dominance, combinatorial multiobjective optimization.

1 Introduction

Evolutionary multiobjective optimization (EMO) has been a hot research area in the field of evolutionary computation for the last two decades [2], [3], [24]. Whereas a large number of various EMO algorithms were proposed, Pareto dominance-based algorithms such as NSGA-II [5], SPEA [29] and SPEA2 [28] have always been the main stream in the EMO community since Goldberg's suggestion [7]. However, the use of scalarizing function-based algorithms (e.g., MOEA/D [27]) and indicator-based algorithms (e.g., SMS-EMOA [1]) have also been actively examined in recent studies, especially for difficult multiobjective problems with complicated Pareto fronts [20] and many-objective problems [25].

In multiobjective optimization, the ranges of values of each objective can be totally different. Those objective values are often normalized in the application of EMO algorithms to multiobjective problems so that the range of values of each objective becomes the same over all objectives. For example, a normalization mechanism was included in the crowding distance calculation of NSGA-II [5]. The importance of the normalization of objective values is widely recognized in the EMO community. This is because almost all elements of EMO algorithms except for Pareto dominance (e.g., crowding mechanisms, hypervolume calculations and scalarizing functions) depend on the magnitude of objective values of each objective.

Objective values are discrete in combinatorial multiobjective optimization due to the combinatorial nature of decision variables. The number of possible values of each objective is totally different. For example, in pattern and feature selection for nearest neighbor classifier design [11], the number of patterns usually has more possible values than the number of features. This is because classification problems usually have more patterns than features (e.g., a magic data set in the UCI Machine Learning Repository has 19,020 patterns with 20 features). In multiobjective genetics-based machine learning [12], the total number of antecedent conditions has more possible values than the number of rules. This is because each rule has a different number of antecedent conditions. In multiobjective flowshop scheduling [16], the maximum flow time has more possible values than the maximum tardiness. This is because a large number of different schedules have the same value of the maximum tardiness even when they have different values of the maximum flow time.

As these examples show, each discrete objective has a different number of possible values (i.e., different granularity). Some discrete objectives have fine granularities with many possible values while others have coarse granularities with only a small number of possible values. We have various examples of multiobjective problems where discrete objectives have totally different granularities. The handling of discrete objectives with different granularities, however, has not been actively studied for EMO algorithms. In our former work [15], we examined the effect of discrete objectives with different granularities on the search behavior of EMO algorithms through computational experiments on two-objective problems. For example, when two objectives had coarse granularities, the search by EMO algorithms was severely slowed down in comparison with the case of two objectives with fine granularities. When two objectives had different granularities, the search was biased towards one objective with a finer granularity. That is, a population was biased towards the edge of the Pareto front with the best value of that objective. The search along the other objective with a coarser granularity was severely slowed down. Whereas we clearly reported those interesting observations in our former work [15], we could not explain why the search by EMO algorithms was affected in such an interesting manner. The main aim of this paper is to explain the reasons for the above-mentioned observations.

This paper is organized as follows. In Section 2, we briefly show the above-mentioned interesting observations in our former work [15]. In Section 3, we clearly explain why those interesting observations were obtained. Based on the explanations in Section 3, we suggest the modification of NSGA-II by the use of strong Pareto dominance in Section 4. It is shown that the suggested modification improves the search ability of NSGA-II on two-objective problems with coarse granularities. In Section 5, we examine the performance of EMO algorithms on many-objective problems with discrete objectives. Experimental results show that discrete objectives with coarse granularities improve the performance of NSGA-II and SPEA2 on many-objective problems while they severely deteriorated the performance on two-objective problems. We also discuss why discrete objectives with coarse granularities have such a positive effect on many-objective optimization from the following two viewpoints: One is the difficulty of many-objective problems for Pareto dominance-based EMO algorithms, and the other is the relation between discrete objectives and the concept of ε-dominance [19]. In Section 6, we conclude this paper.

2 Effect of Discrete Objectives on Two-Objective Optimization

In our former work [15], we examined the effect of discrete objectives with different granularities on the search behavior of NSGA-II [5], SPEA2 [28], MOEA/D [27] and SMS-EMOA [1] on the following four types of two-objective problems:

(i) Two-objective 500-item knapsack problem in Zitzler and Thiele [29],
(ii) 500-bit one-max and zero-max problem,
(iii) Modified 500-bit one-max and zero-max problem with a convex Pareto front,
(iv) Modified 500-bit one-max and zero-max problem with a concave Pareto front.

Similar effects of discrete objectives were observed on the search behavior of the four EMO algorithms on the three types of two-objective problems in our former work. Here we only show experimental results of NSGA-II on the two-objective 500-item knapsack problem in Zitzler and Thiele [29].

The two-objective 500-item knapsack problem with two constraint conditions in Zitzler and Thiele [29] is written as follows:

$$\text{Maximize} \quad f_i(\boldsymbol{x}) = \sum_{j=1}^{n} p_{ij} x_j , \quad i = 1, 2, \tag{1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} w_{ij} x_j \leq c_i , \quad i = 1, 2, \tag{2}$$

$$x_j = 0 \text{ or } 1, \quad j = 1, 2, \ldots, n. \tag{3}$$

In (1)-(3), n is the number of items (i.e., n = 500 in this paper), $\boldsymbol{x}$ is a 500-bit binary string, p_{ij} is the profit of item j according to knapsack i, w_{ij} is the weight of item j according to knapsack i, and c_i is the capacity of knapsack i. The value of each profit p_{ij} in (1) was randomly specified as an integer in the interval [10, 100]. As a result, each objective has integer objective values. We use exactly the same two-objective 500-item knapsack problem as in Zitzler and Thiele [29].

In Fig. 1, we show randomly generated 200 solutions of this two-objective 500-item knapsack problem together with its Pareto front. In Fig. 1, we used a greedy repair method based on the maximum profit/weight ratio in Zitzler and Thiele [29] when randomly generated solutions were infeasible. The greedy repair method in [29] was always used in our computational experiments in this paper. As shown in Fig. 1, randomly generated solutions are not close to the Pareto front. Thus a high selection pressure towards the Pareto front is needed to efficiently search for Pareto optimal or near Pareto optimal solutions. At the same time, a strong diversity improvement mechanism is also needed to find a wide variety of solutions along the entire Pareto front. That is, EMO algorithms for the knapsack problem in Fig. 1 need strong convergence and diversification properties. Multiobjective knapsack problems have been frequently used to evaluate the performance of EMO algorithms in the literature (e.g., Jaszkiewicz [17] and Sato et al. [22]).

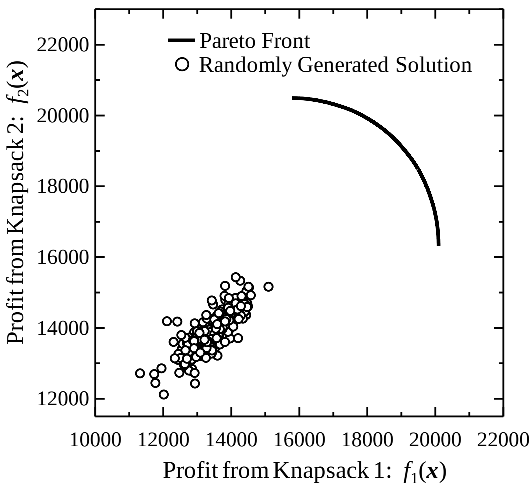


Fig. 1. Pareto front and randomly generated 200 solutions [15].

In our former work [15], NSGA-II, SPEA2, MOEA/D and SMS-EMOA with the following parameter specifications were applied to the knapsack problem in Fig. 1:

Coding: Binary string of length 500 (i.e., 500-bit string),
Population size: 200,
Termination condition: 2000 generations (400000 solution evaluations in MOEA/D),
Parent selection: Random selection from the population (SMS-EMOA),
Random selection from the neighborhood (MOEA/D),
Binary tournament selection with replacement (NSGA-II and SPEA2),
Crossover: Uniform crossover (Probability: 0.8),
Mutation: Bit-flip mutation (Probability: 1/500),
Number of runs for each test problem: 100 runs.

The origin (0, 0) of the two-dimensional objective space was used as a reference point for hypervolume calculation in SMS-EMOA. In MOEA/D, the weighted

Tchebycheff function was used in the same manner as in Zhang and Li [27]. The neighborhood size in MOEA/D was specified as 10.

The four EMO algorithms were also applied to discretized problems with the discretization interval of width 100. For example, objective values in [20001, 20100] and [20101, 20200] were rounded up to 20100 and 20200, respectively. It should be noted that the width of the discretization interval for each objective in the original knapsack problem in Fig. 1 is 1. This is because each profit p_{ij} in the two objective functions was randomly specified as an integer in the interval [10, 100]. We denote discretized problems using the width of the discretization interval for each objective such as G100-G1 and G100-G100. In the G100-G100 problem, both objectives were discretized by the discretization interval of width 100. The original knapsack problem is denoted as G1-G1. Only the first objective of G100-G1 (only the second objective of G1-G100) was discretized by the discretization interval of width 100.

In Fig. 2, we show experimental results by NSGA-II on the four knapsack problems (i.e., G1-G1, G1-G100, G100-G1 and G100-G100). In each plot of Fig. 2, all solutions at the final generation of a single run of NSGA-II are shown together with the 50% attainment surface [6] over its 100 runs.

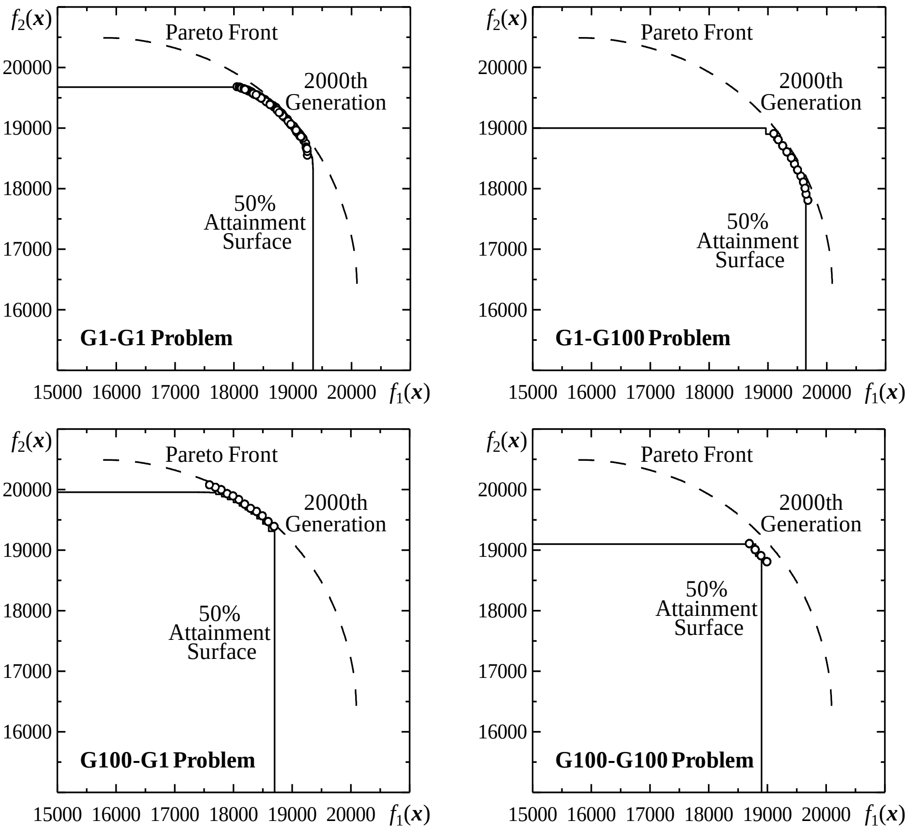


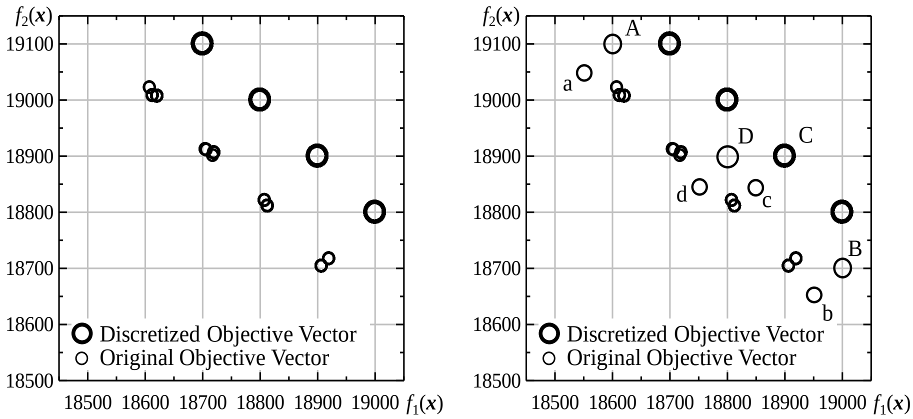
Fig. 2. Experimental results by NSGA-II on four variants of the knapsack problem.

3 Discussions on the Search Behavior of EMO Algorithms

From each plot in Fig. 2, we can see that the following results were obtained about the search behavior of NSGA-II on each problem:

(i) G1-G1: NSGA-II found many solutions around the center of the Pareto front.
(ii) G1-G100: The search was biased towards the bottom-right part of the Pareto front.
(iii) G100-G1: The search was biased towards the top-left part of the Pareto front.
(iv) G100-G100: The performance of NSGA-II was severely deteriorated.

Let us discuss why these results were obtained. First we address the search behavior of NSGA-II on the G100-G100 problem. In the bottom-right plot of Fig. 2, only four solutions of the G100-G100 problem were obtained by a single run of NSGA-II. We checked all the 200 solutions in the final population. Then we found that they were overlapping on the four solutions in the discretized objective space. We also found that the number of different strings in the final population was eleven. All of them had different objective vectors in the original objective space (i.e., the objective space of the G1-G1 problem). The four solutions in the discretized objective space and the eleven solutions in the original objective space are shown in an enlarged view in the left plot of Fig. 3.



(a) Actually obtained solutions. (b) Solutions for illustration purposes.

Fig. 3. Obtained Solutions by a single run of NSGA-II on the G100-G100 problem.

Using the right plot of Fig. 3, we explain why the search ability of NSGA-II on the G100-G100 problem was severely deteriorated. Let us assume that new solutions “a” and “b” are generated by crossover and mutation. Whereas those solutions increase the diversity, they are not likely to survive because they are dominated solutions in the discretized objective space (i.e., because solutions “A” and “B” are dominated solutions). This explains why the diversity of obtained solutions for the G100-G100 problem was very small in Fig. 2. Let us also assume that a new solution “c” is generated by crossover and mutation. Whereas this solution is better than the two solutions in the same box, all of them are discretized to the same solution “C” in the discretized objective space. Thus the difference between the new solution “c” and the existing two solutions in the same box disappears by the discretization. This explains why the search of NSGA-II towards the Pareto front of the G100-G100 problem was slow in Fig. 2. Further we assume that a new solution “d” in the right plot of Fig. 3 is generated by crossover and mutation. This solution is not likely to survive because its discretized solution “D” is a dominated solution. The situation of the new solution “d” also explains the deterioration in the search ability of NSGA-II.

Next, let us address the search behavior of NSGA-II on the G100-G1 problem. The first objective of this problem has a very coarse granularity (i.e., G100) while its second objective is a fine granularity (i.e., G1). The objective space of the G100-G1 problem is illustrated in Fig. 4. Ten solutions in Fig. 4 are non-dominated with each other in the objective space of the original G1-G1 problem. However, six of them are dominated solutions in the objective space of the G100-G1 problem. For example, let us consider the four solutions “e”, “f”, “g” and “h” in the objective space of the original G1-G1 problem in Fig. 4. They are discretized to the solutions “E”, “F”, “G” and “H” in the objective space of the G100-G1 problem, respectively. Whereas “e”, “f”, “g” and “h” are non-dominated with each other, “F”, “G” and “H” are dominated by “E”. In the fitness evaluation of NSGA-II, the ranks of these solutions are as follows: E: Rank 1, F: Rank 2, G: Rank 3, H: Rank 4. Thus the solutions G and H are likely to be removed in the generation update phase of NSGA-II. This explains why the multiobjective search of NSGA-II on the G100-G1 problem was biased towards the top-left part of the Pareto front in Fig. 2. In the same manner, we can explain why the multiobjective search of NSGA-II on the G1-G100 problem was biased towards the bottom-right part of the Pareto front in Fig. 2.

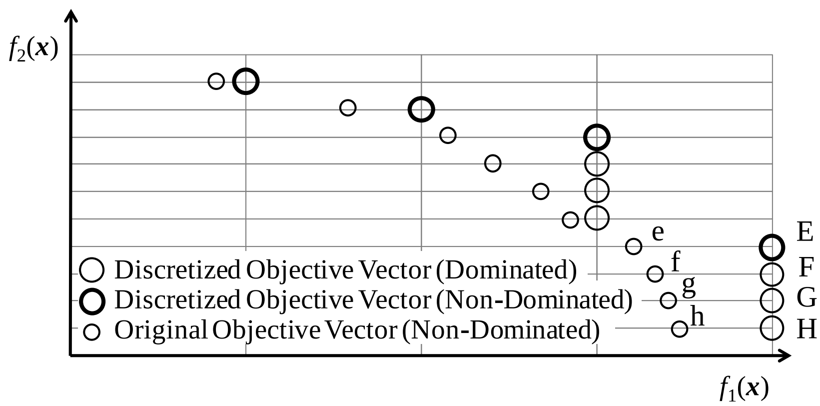


Fig. 4. Explanations of the search behavior by NSGA-II on the G100-G1 problem.

4 Use of Strong Pareto Dominance in NSGA-II

As explained in Section 3, many non-dominated solutions of the original G1-G1 problem become dominated solutions by the use of the coarse granularity G100. For example, the non-dominated solutions “f”, “g” and “h” in Fig. 4 were discretized to the dominated solutions "F", "G" and "H" by the use of G100 for the first objective. In the right plot in Fig. 3, the non-dominated solutions "a", "b" and "d" were discretized to the dominated solutions "A", "B" and "D". Such a solution status change seems to have a lot of negative effects on the search ability of NSGA-II. In other words, the handling of those dominated solutions as non-dominated ones may prevent the deterioration in the performance of NSGA-II.

Motivated by these discussions, let us examine the modification of NSGA-II by using the following strong Pareto dominance in the fitness evaluation of NSGA-II:

Strong Pareto Dominance

For multiobjective maximization, an objective vector $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), ..., f_n(\boldsymbol{x}))$ is defined as being strongly dominated by another objective vector $\boldsymbol{f}(\boldsymbol{y}) = (f_1(\boldsymbol{y}), f_2(\boldsymbol{y}), ..., f_n(\boldsymbol{y}))$ if and only if $f_i(\boldsymbol{x}) < f_i(\boldsymbol{y})$ holds for all $i = 1, 2, ..., n$.

When we use this definition, solutions "A", "B", "D", "F", "G" and "H" in Fig. 3 and Fig. 4 are handled as non-dominated solutions. We applied NSGA-II with this definition to the G1-G1, G1-G100, G100-G1 and G100-G100 problems in the same manner as in Section 2. Experimental results are shown in Fig. 5.

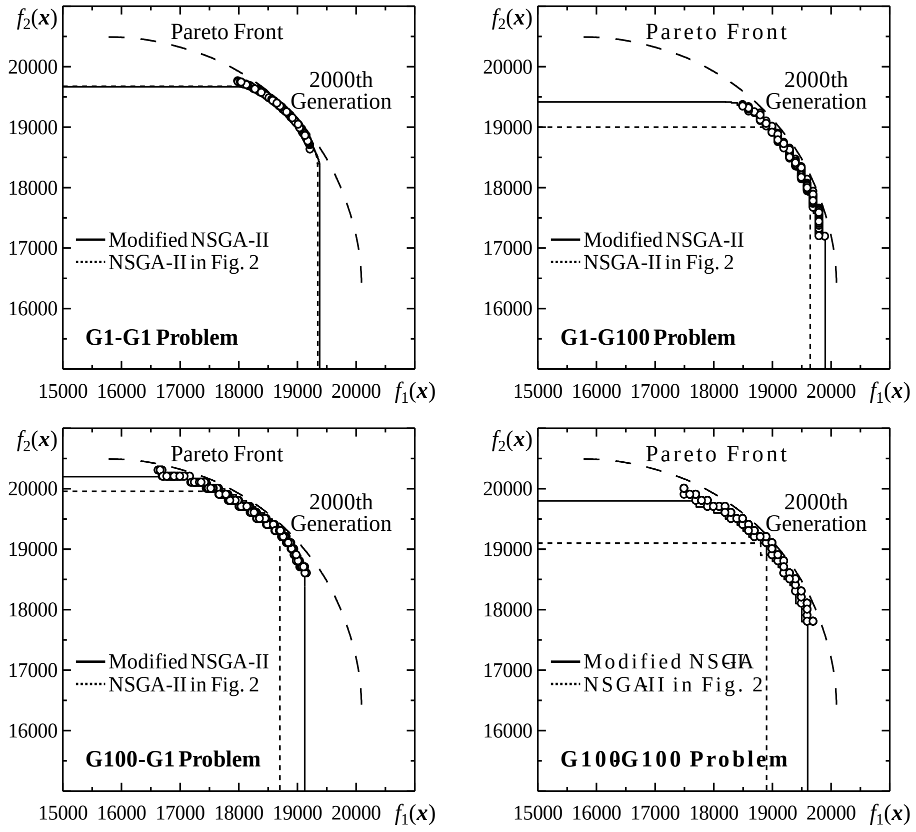


Fig. 5. Experimental results by NSGA-II with strong Pareto dominance.

From the comparison between Fig. 2 and Fig. 5, we can obtain the following observations about the search ability of NSGA-II:

(i) The use of strong Pareto dominance had almost no effect on the search ability of NSGA-II on the G1-G1 problem (see the top-left plot of each figure). This is because the difference between the standard and modified NSGA-II algorithms is very small when the granularity is fine (i.e., because $f_i(\boldsymbol{x}) = f_i(\boldsymbol{y})$ is not likely to hold for many pairs of different solutions $\boldsymbol{x}$ and $\boldsymbol{y}$ when the granularity is fine).

(ii) The search ability of NSGA-II on the G100-G100 problem was clearly improved by the use of strong Pareto dominance with respect to both the convergence and the diversity (see the bottom-right plot of each figure). That is, the intended positive effect of strong Pareto dominance was observed for the G100-G100 problem.

(iii) The search ability of NSGA-II on the G1-G100 and G100-G1 problems was also clearly improved by the use of strong Pareto dominance with respect to the diversity of obtained solutions. The bias of the search of NSGA-II toward a part of the Pareto front was somewhat remedied by the use of strong Pareto dominance. That is, the intended positive effect of strong Pareto dominance was also observed for the G1-G100 and G100-G1 problems.

5 Computational Experiments on Many-Objective Problems

We have already examined the effect of discrete objectives with different granularities on the search behavior of NSGA-II on the four two-objective knapsack problems. We have also demonstrated that the use of strong Pareto dominance improved the performance of NSGA-II on the three two-objective knapsack problems with the coarse granularity. In this section, we examine the performance of NSGA-II, SPEA2, MOEA/D and SMS-EMOA on many-objective knapsack problems with fine and coarse granularities using the hypervolume measure.

Many-objective optimization with four or more objectives is usually very difficult for Pareto dominance-based EMO algorithms [10], [18], [21]. This is because almost all solutions in a population quickly become non-dominated within a small number of generations in evolutionary multiobjective search for many-objective problems. As a result, Pareto dominance-based selection pressure quickly becomes very weak. Various approaches have been proposed to improve the search ability of Pareto dominance-based EMO algorithms on many-objective problems [13], [14]. Recently it has also been shown that many-objective problems are not necessarily difficult for Pareto dominance-based EMO algorithms in the literature [23].

As test problems, we generated multiobjective 500-item knapsack problems with up to eight objectives by adding randomly generated objectives $f_i(\boldsymbol{x})$, i = 3, 4, ..., 8 to the original two-objective 500-item knapsack problems in the previous sections:

$$f_i(\boldsymbol{x}) = \sum_{j=1}^{500} p_{ij} x_j , \quad i = 3, 4, ..., 8, \tag{4}$$

where p_{ij} is a randomly specified integer in the interval [10, 100].

Using the same parameter specifications as in Section 2, we applied NSGA-II, its modified version, SPEA2, MOEA/D and SMS-EMOA to our test problems with two, four, six and eight objectives. Only in MOEA/D, the population size was specified as 220, 252 and 120 for our test problems with four, six and eight objectives, respectively. This is due to the combinatorial nature in the number of weight vectors in MOEA/D (for details, see [27]). The population size was specified as 200 in all the other cases. We used a fast hypervolume calculation method by While et al. [26] in SMS-EMOA. Each algorithm was applied to each test problem 100 times to calculate the average hypervolume except for SMS-EMOA on the eight-objective problem due to its heavy computation load: 30 runs of SMS-EMOA on the eight-objective problem.

Each test problem was discretized using the discretization intervals of width 10 (i.e., granularity: G10) and width 100 (i.e., granularity: G100). The granularity of our test problems before the discretization was G1 since each profit value in the objective functions was randomly specified as an integer in the interval [10, 100]. We used the origin of the objective space as a reference point for hypervolume calculation. The hypervolume calculation was always performed in the original objective space with the granularity G1. That is, discretized objective values with the granularities G10 and G100 were restored to their original values with G1 for fair comparison.

In Tables 1-5, we summarize our experimental results by each EMO algorithm. Each table shows the average hypervolume value and the standard deviation by each EMO algorithm on each test problem. The best result (i.e., the largest average hypervolume) among the three granularity specifications is highlighted by boldface for each test problem in each table.

Now, let us examine experimental results in each table. In Table 1, experimental results by NSGA-II are summarized. As we have already demonstrated in Fig. 2 in Section 2, the use of the coarse granularity G100 deteriorated the performance of NSGA-II on the two-objective problem in Table 1 (i.e., 5.6% decrease in the average hypervolume from 3.800×10^8 to 3.588×10^8). However, it improved the performance of NSGA-II on the eight-objective problem by 3.0% from 1.100×10^{34} to 1.133×10^{34}.

Let us discuss these observations from the viewpoint of ε-dominance which was proposed by Laumanns et al. [19] and used in ε-MOEA [4]. The comparison between boxes in the discretized objective space in ε-MOEA [4] is the same as the Pareto dominance-based comparison between discretized objective vectors in this paper. Horoba and Neumann [8], [9] theoretically explained that the use of ε-dominance deteriorates the search ability of their EMO algorithm when many Pareto-optimal solutions are included in a single box. This may be related to the performance deterioration by the use of G100 for the two-objective problem in Table 1 because a large number of Pareto-optimal solutions are included in a single box with G100.

Table 1. Average hypervolume and standard deviation by NSGA-II.

Problem	Granularity: G1		Granularity: G10		Granularity: G100	
	Average	Stand. Dev.	Average	Stand. Dev.	Average	Stand. Dev.
2-Objective	**3.800E+08**	1.703E+06	3.779E+08	1.416E+06	3.588E+08	1.797E+06
4-Objective	1.227E+17	0.925E+15	**1.235E+17**	0.918E+15	1.209E+17	1.306E+15
6-Objective	3.729E+25	3.885E+23	3.751E+25	4.065E+23	**3.813E+25**	4.149E+23
8-Objective	1.100E+34	1.722E+32	1.112E+34	1.452E+32	**1.133E+34**	1.475E+32

Horoba and Neumann [8], [9] also proved that the use of ε-dominance improves the search ability of their EMO algorithm when the number of Pareto-optimal solutions exponentially increases with the problem size. This may be related to the performance improvement by the use of G100 for the six-objective and eight-objective problems in Table 1. The performance improvement can be also explained by the decrease in the number of non-dominated solutions. As we demonstrated in Section 3 using Fig. 3, many non-dominated solutions become dominated after the discretization of objective values. This means that the number of non-dominated solutions is decreased by the discretization. Since the difficulty of many-objective problems is caused by the increase in the number of non-dominated solutions, the discretization of objective values is likely to work as a countermeasure for improving the performance of NSGA-II on many-objective problems.

Whereas we have explained the effect of discrete objectives using the concept of ε-dominance, there is a clear difference between the discretization in this paper and the use of ε-dominance. In this paper, we assume that each objective has discrete values. For example, a discrete objective with the granularity G100 is assumed to be measured in multiples of 100. Thus its objective values are always multiples of 100 such as 500 and 600. However, the objective space discretization by ε-dominance is used only for the comparison between boxes. Objective values are not discretized. Thus EMO algorithms based on ε-dominance can use the standard Pareto dominance between objective vectors as well as ε-dominance between boxes. For example, two objective vectors (525, 550) and (531, 566) can be compared even when the objective space is discretized by ε-dominance using the discretization interval of width 100. This is not the case in this paper because these two objective vectors are always handled as the same objective vector (600, 600) in the case of the granularity G100. That is, they cannot be compared in this paper when the granularity is G100.

Table 2 shows experimental results by NSGA-II with strong Pareto dominance. As we have already demonstrated in Fig. 5, the use of strong Pareto dominance improved the performance of NSGA-II on the two-objective problem with G100 from Table 1 to Table 2 (i.e., 7.2% increase from 3.588×10^8 in Table 1 to 3.843×10^8 in Table 2). However, it deteriorated the performance of NSGA-II on the eight-objective problem with G100 by 4.3% from 1.133×10^{34} in Table 1 to 1.084×10^{34} in Table 2. This is because the use of strong Pareto dominance increases the number of non-dominated solutions, which is the main reason for the difficulty in the handling of many-objective problems by Pareto dominance-based EMO algorithms. That is, the increase in the number of non-dominated solutions by the use of strong Pareto dominance has a positive effect on the two-objective problem and a negative effect on the eight-objective problem.

Table 2. Average hypervolume and standard deviation by the modified NSGA-II.

Problem	Granularity: G1		Granularity: G10		Granularity: G100	
	Average	Stand. Dev.	Average	Stand. Dev.	Average	Stand. Dev.
2-Objective	3.806E+08	1.603E+06	3.819E+08	1.419E+06	**3.843E+08**	1.925E+06
4-Objective	1.228E+17	1.016E+15	1.230E+17	0.841E+15	**1.290E+17**	1.032E+15
6-Objective	3.728E+25	4.020E+23	3.726E+25	4.078E+23	**3.795E+25**	4.276E+23
8-Objective	1.093E+34	1.924E+32	**1.097E+34**	1.793E+32	1.084E+34	2.046E+32

Table 3 shows experimental results by SPEA2. We can obtain similar observations from Table 1 (NSGA-II) and Table 3 (SPEA2). That is, the discretization of objective values by the granularity G100 deteriorated the performance of SPEA2 on the two-objective problem and improved its performance on the eight-objective problem in Table 3. This is because fitness evaluation in NSGA-II and SPEA2 is based on Pareto dominance (i.e., because they are Pareto dominance-based EMO algorithms).

Table 3. Average hypervolume and standard deviation by SPEA2.

Problem	Granularity: G1		Granularity: G10		Granularity: G100	
	Average	Stand. Dev.	Average	Stand. Dev.	Average	Stand. Dev.
2-Objective	**3.786E+08**	1.162E+06	3.777E+08	1.657E+06	3.623E+08	2.098E+06
4-Objective	**1.218E+17**	0.801E+15	1.218E+17	0.889E+15	1.161E+17	1.148E+15
6-Objective	3.553E+25	4.411E+23	3.558E+25	3.863E+23	**3.619E+25**	3.600E+23
8-Objective	1.029E+34	1.362E+32	1.032E+34	1.396E+32	**1.068E+34**	1.295E+32

Table 4 shows experimental results by MOEA/D. We can observe totally different effects of discrete objectives in Table 4 on MOEA/D from Table 1 on NSGA-II and Table 3 on SPEA2. That is, the use of the coarse granularities G10 and G100 monotonically deteriorated the performance of MOEA/D on all the test problems with 2-8 objectives in Table 4. This is because the discretization of objective values simply makes single-objective optimization of scalarizing functions difficult independent of the number of objectives in MOEA/D.

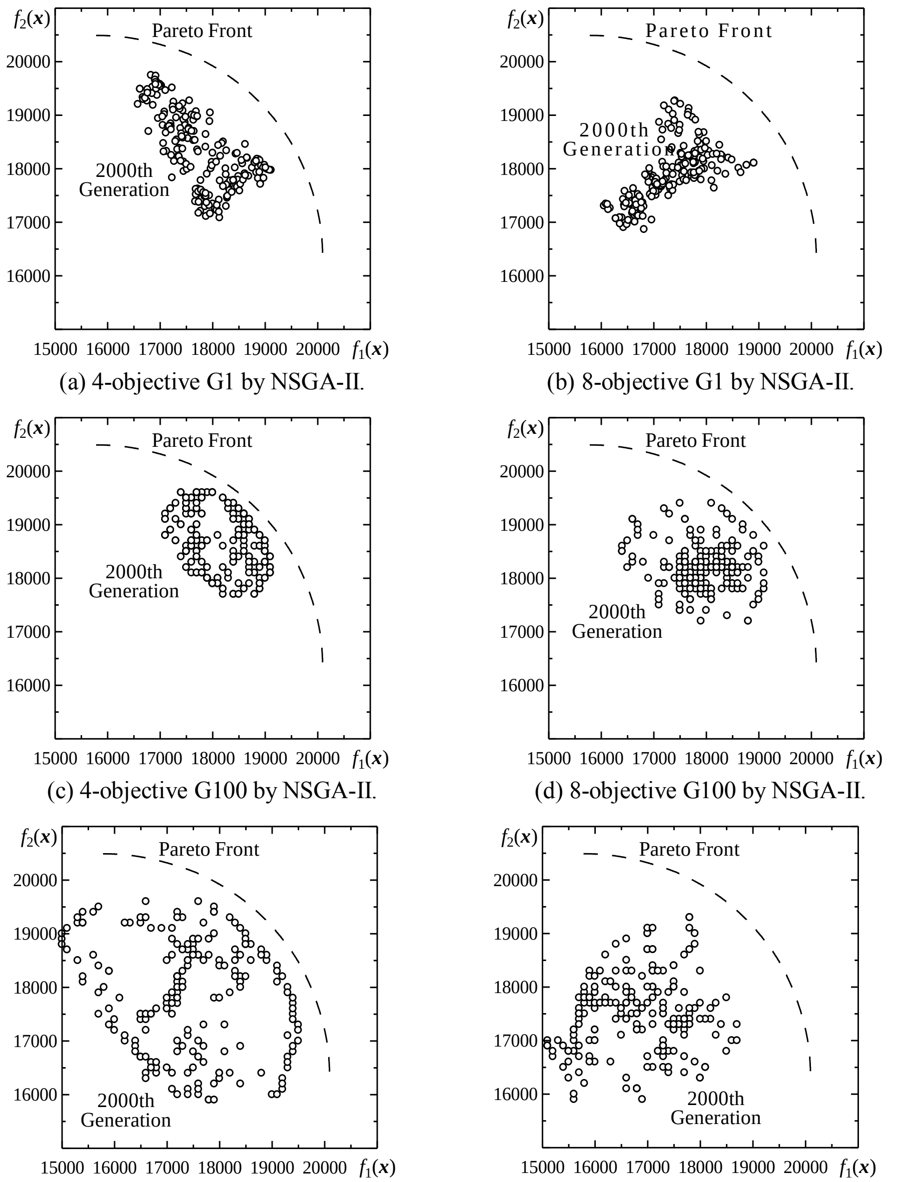
Table 4. Average hypervolume and standard deviation by MOEA/D.

Problem	Granularity: G1		Granularity: G10		Granularity: G100	
	Average	Stand. Dev.	Average	Stand. Dev.	Average	Stand. Dev.
2-Objective	**4.009E+08**	0.931E+06	3.993E+08	1.034E+06	3.610E+08	4.382E+06
4-Objective	**1.430E+17**	0.705E+15	1.421E+17	0.745E+15	1.249E+17	2.114E+15
6-Objective	**4.525E+25**	3.778E+23	4.484E+25	3.912E+23	3.763E+25	9.70E+23
8-Objective	**1.355E+34**	1.335E+32	1.340E+34	1.563E+32	1.022E+34	2.191E+32

Table 5 shows experimental results by SMS-EMOA. Effects of discrete objectives on SMS-EMOA in Table 5 are similar to those on MOEA/D in Table 4 (i.e., the discretization of objective values deteriorated the performance of SMS-EMOA). This may be because their fitness evaluation is not based on Pareto dominance.

Table 5. Average hypervolume and standard deviation by SMS-EMOA.

Problem	Granularity: G1		Granularity: G10		Granularity: G100	
	Average	Stand. Dev.	Average	Stand. Dev.	Average	Stand. Dev.
2-Objective	**3.760E+08**	1.734E+06	3.730E+08	2.042E+06	3.571E+08	1.695E+06
4-Objective	**1.285E+17**	8.128E+14	1.277E+17	9.275E+14	1.129E+17	2.356E+15
6-Objective	**4.146E+25**	3.322E+23	4.141E+25	3.611E+23	3.784E+25	3.845E+23
8-Objective	1.305E+34	1.227E+32	**1.309E+34**	1.400E+32	1.205E+34	1.292E+32



(a) 4-objective G1 by NSGA-II. (b) 8-objective G1 by NSGA-II.

(c) 4-objective G100 by NSGA-II. (d) 8-objective G100 by NSGA-II.

(e) 4-objective G100 by modified NSGA-II. (f) 8-objective G100 by modified NSGA-II.

Fig. 6. Projection of solutions in the final generation of NSGA-II and the modified NSGA-II.

Fig. 6 shows the projection of a final population on the two-dimensional subspace with $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ in a single run of NSGA-II and its modified version on the four-objective and eight-objective problem with G1 and G100. Fig. 6 shows the increase in the diversity of solutions and the deterioration in their convergence by the use of strong Pareto dominance for many-objective problems with G100.

6 Conclusions

In this paper, we first explained why discrete objectives with coarse granularities deteriorated the search ability of EMO algorithms on two-objective problems. Next we proposed the use of strong Pareto dominance, which improved the performance of NSGA-II on discrete two-objective problems with coarse granularities. Then we examined the effect of discrete objectives on the performance of EMO algorithms on many-objective problems. Finally we discussed why the use of coarse granularities improved the performance of NSGA-II and SPEA2 on many-objective problems whereas it deteriorated the performance of MOEA/D and SMS-EMOA. The performance improvement of NSGA-II and SPEA2 on many-objective problems was explained from the difficulty of many-objective problems (i.e., the increase in the number of non-dominated solutions). Since the use of coarse granularities decreases the number of non-dominated solutions, it remedies the difficulty of many-objective problems for Pareto dominance-based EMO algorithms. We also discussed the effect of discrete objectives on the performance of EMO algorithms from the viewpoint of the concept of ε-dominance. Our observations were compared with some theoretical studies [8], [9]. It is an interesting future research topic to examine the performance of ε-dominance EMO algorithms on discrete many-objective problems with different granularities in comparison with Pareto dominance-based EMO algorithms.

References

1. Beume, N., Naujoks, B., Emmerich, M.: SMS-EMOA: Multiobjective Selection based on Dominated Hypervolume. European Journal of Operational Research 181 (2007) 1653-1669
2. Coello, C. A. C., Lamont, G. B.: Applications of Multi-Objective Evolutionary Algorithms. World Scientific, Singapore (2004)
3. Deb, K.: Multi-Objective Optimization Using Evolutionary Algorithms. John Wiley & Sons, Chichester (2001)
4. Deb, K., Mohan, M., Mishra, S.: Evaluating the ε-Domination Based Multi-Objective Evolutionary Algorithm for a Quick Computation of Pareto-Optimal Solutions. Evolutionary Computation 13 (2005) 501-525
5. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. IEEE Trans. on Evolutionary Computation 6 (2002) 182-197
6. Fonseca, C. M., Fleming, P. J.: On the Performance Assessment and Comparison of Stochastic Multiobjective Optimizers. Lecture Notes in Computer Science, Vol. 114: PPSN IV. Springer, Berlin (1996) 584-593
7. Goldberg, D. E.: Genetic Algorithms in Search, Optimization, and Machine Learning. Addison-Wesley, Reasing (1989)
8. Horoba, C., Neumann, F.: Benefits and Drawbacks for the Use of ε-Dominance in Evolutionary Multi-Objective Optimization. Proc. of 2008 Genetic and Evolutionary Computation Conference (2008) 641-648
9. Horoba, C., Neumann, F.: Additive Approximations of Pareto-Optimal Sets by Evolutionary Multi-Objective Algorithms. Proc. of 10th ACM SIGEVO Workshop on Foundations of Genetic Algorithms (2009) 79-86
10. Hughes, E. J.: Evolutionary Many-Objective Optimization: Many Once or One Many?. Proc. of 2005 IEEE Congress on Evolutionary Computation (2005) 222-227

11. Ishibuchi, H., Nakashima, T.: Multi-Objective Pattern and Feature Selection by a Genetic Algorithm. Proc. of 2000 Genetic and Evolutionary Computation Conference (2000) 1069-1076
12. Ishibuchi, H., Nojima, Y.: Analysis of Interpretability-Accuracy Tradeoff of Fuzzy Systems by Multiobjective Fuzzy Genetics-Based Machine Learning. International Journal of Approximate Reasoning 44 (2007) 4-31
13. Ishibuchi, H., Tsukamoto, N., Hitotsuyanagi, Y., Nojima, Y.: Effectiveness of Scalability Improvement Attempts on the Performance of NSGA-II for Many-Objective Problems. Proc. of 2008 Genetic and Evolutionary Computation Conference (2008) 649-656
14. Ishibuchi, H., Tsukamoto, N., Nojima, Y.: Evolutionary Many-Objective Optimization: A Short Review. Proc. of 2008 IEEE Congress on Evolutionary Computation (2008) 2424-2431
15. Ishibuchi, H., Yamane, M., Nojima, Y.: Effects of Discrete Objective Functions with Different Granularities on the Search Behavior of EMO Algorithms, Proc. of 2012 Genetic and Evolutionary Computation Conference (2012) 481-488
16. Ishibuchi, H., Yoshida, T., Murata, T.: Balance between Genetic Search and Local Search in Memetic Algorithms for Multiobjective Permutation Flowshop Scheduling. IEEE Trans. on Evolutionary Computation 7 (2003) 204-223
17. Jaszkiewicz, A.: On the Computational Efficiency of Multiple Objective Metaheuristics: The Knapsack Problem Case Study. European Journal of Operational Research 158 (2004) 418-433
18. Khare, V., Yao, X., Deb, K.: Performance Scaling of Multi-Objective Evolutionary Algorithms. Lecture Notes in Computer Science, Vol. 2632: EMO 2003, Springer, Berlin (2004) 367-390
19. Laumanns, M., Thiele, L., Deb, K., Zitzler, E.: Combining Convergence and Diversity in Evolutionary Multiobjective Optimization. Evolutionary Computation 10 (2002) 263-282
20. Li, H., Zhang, Q.: Multiobjective Optimization Problems with Complicated Pareto Sets, MOEA/D and NSGA-II. IEEE Trans. on Evolutionary Computation 13 (2009) 284-302
21. Purshouse, R. C., Fleming, P. J.: On the Evolutionary Optimization of Many Conflicting Objectives. IEEE Trans. on Evolutionary Computation 11 (2007) 770-784
22. Sato, H., Aguirre, H. E., Tanaka, K.: Controlling Dominance Area of Solutions and Its Impact on the Performance of MOEAs. Lecture Notes in Computer Science, Vol. 4403: EMO 2007, Springer, Berlin (2007) 5-20
23. Schütze, O., Lara, A., Coello, C. A. C.: On the Influence of the Number of Objectives on the Hardness of a Multiobjective Optimization Problem. IEEE Trans. on Evolutionary Computation 15 (2011) 444-455
24. Tan, K. C., Khor, E. F., Lee, T. H.: Multiobjective Evolutionary Algorithms and Applications. Springer, Berlin (2005)
25. Wagner, T., Beume, N., Naujoks, B.: Pareto-, Aggregation-, and Indicator-Based Methods in Many-Objective Optimization. Lecture Notes in Computer Science, Vol. 4403: EMO 2007, Springer, Berlin (2007) 742-756
26. While, L., Bradstreet, L., Barone, L.: A Fast Way of Calculating Exact Hypervolumes. IEEE Trans. on Evolutionary Computation 16 (2012) 86-95
27. Zhang, Q., Li, H.: MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. IEEE Trans. on Evolutionary Computation 11 (2007) 712-731
28. Zitzler, E., Laumanns, M., Thiele L.: SPEA2: Improving the Strength Pareto Evolutionary Algorithm. TIK-Report 103, Computer Engineering and Networks Laboratory (TIK), Department of Electrical Engineering, ETH, Zurich (2001)
29. Zitzler, E., Thiele, L.: Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach. IEEE Trans. on Evolutionary Computation 3 (1999) 257-271