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Single-Objective and Multi-Objective Formulations of Solution Selection for Hypervolume Maximization

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ABSTRACT

A new trend in evolutionary multi-objective optimization (EMO) is the handling of a multi-objective problem as an optimization problem of an indicator function. A number of approaches have been proposed under the name of indicator-based evolutionary algorithms (IBEAs). In IBEAs, the entire population usually corresponds to a solution of the indicator optimization problem. In this paper, we show how hypervolume maximization can be handled as single-objective and multi-objective problems by coding a set of solutions of the original multi-objective problem as an individual. Our single-objective formulation maximizes the hypervolume under constraint conditions on the number of non-dominated solutions. On the other hand, our multi-objective formulation minimizes the number of non-dominated solutions while maximizing their Hypervolume.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Heuristic Methods.*

General Terms

Algorithms.

Keywords

Evolutionary multi-objective optimization, solution selection, indicator-based evolutionary algorithm, solution set optimization.

1. INTRODUCTION

Recently multi-objective problems have been handled as indicator optimization problems [1]-[5], [7]. Those approaches are often referred to as indicator-based evolutionary algorithms (IBEAs). The hypervolume measure is frequently used as an indicator function in IBEAs. Such an indicator function is used to evaluate solution sets instead of solutions. Thus IBEAs can be viewed as single-objective optimizers for finding optimal solution sets.

An individual in IBEAs is a single solution of the original multi-objective problem. In this paper, we handle a set of solutions as an individual for hypervolume maximization. Thus an individual is a solution of the hypervolume maximization problem whereas the entire population corresponds to its single solution in IBEAs. We formulate hypervolume maximization as single-objective and multi-objective problems. The hypervolume is maximized under constraint conditions on the number of non-dominated solutions in our single-objective formulation. On the other hand, our multi-objective formulation minimizes the number of non-dominated solutions while maximizing their hypervolume.

2. PROBLEM FORMULATIONS

Let us consider the following k-objective maximization problem:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})) \text{ subject to } \mathbf{x} \in \mathbf{X}, \tag{1}$$

where $\mathbf{f}(\mathbf{x})$ is the k-dimensional objective vector, $f_i(\mathbf{x})$ is the i-th objective to be maximized ($i = 1, 2, \ldots, k$), $\mathbf{x}$ is the decision vector, and $\mathbf{X}$ is the feasible region in the decision space.

Let S be a set of solutions, which is optimized by IBEAs. The solution set S can be also viewed as a subset of the feasible region $\mathbf{X}$. We denote an indicator function used in IBEAs by $I(S)$, which measures the quality of the solution set S. Whereas we implicitly assume the use of the hypervolume measure in this paper, other indicator functions can be used as $I(S)$ in our formulations. The execution of IBEAs with a fixed population size N_{pop} can be viewed as solving the following maximization problem:

[Single-Objective Formulation with an Equality Condition]

$$\text{Maximize } I(S) \text{ subject to } S \subset \mathbf{X} \text{ and } |S| = N_{\text{pop}}, \tag{2}$$

where $|S|$ is the number of solutions in S.

When the decision maker does not want to see more than N^* non-dominated solutions, N^* should be handled as the upper limit on the number of solutions in the solution set S. In this case, the single-objective formulation in (2) is modified as

[Single-Objective Formulation with an Inequality Condition]

$$\text{Maximize } I(S) \text{ subject to } S \subset \mathbf{X} \text{ and } |S| \leq N^*. \tag{3}$$

Single-objective formulations in (2) and (3) were discussed in [1]. When the decision maker wants to minimize the number of presented non-dominated solutions, one possible formulation is the use of the following weighted sum objective function:

[Weighed Sum-based Single-Objective Formulation]

Maximize $w_1 \cdot I(S) - w_2 \cdot |S|$ subject to $S \subset \mathbf{X}$, (4)

where $\mathbf{w} = (w_1, w_2)$ is a non-negative weight vector.

Of course, we can use other scalarizing functions in (4) instead of the weighted sum. In general, an appropriate specification of a scalarizing function is difficult. In order to avoid the difficulty in the specification of scalarizing functions, we can formulate the following multi-objective problem with two objectives:

[Multi-Objective Formulation]

Maximize $I(S)$ and minimize $|S|$ subject to $S \subset \mathbf{X}$. (5)

3. IMPLEMENTATION OF OUR IDEA

It is possible to implement general IBEAs by coding a solution set S using a concatenated string "$\mathbf{x}_1 \mathbf{x}_2 \ldots \mathbf{x}_m$" of variable string length. In this coding, $\mathbf{x}_i$ $(i = 1, 2, \ldots, m)$ is a solution of the original multi-objective problem in (1) and m is the number of solutions included in the solution set S. IBEAs with such a concatenated string of variable string length, however, may need a large computation load and/or a number of sophisticated speed-up tricks to efficiently search for good solution sets. This is because the search space is huge. We leave the implementation of general IBEAs and their performance evaluation as future research issues. In this paper, we show an IBEA implementation for a very special case where a large number of non-dominated solutions have already been given. The use of such a special case is to clearly demonstrate the usefulness of our basic idea: to handle solution sets as individuals in IBEAs.

Let us assume that N non-dominated solutions ($\mathbf{x}_1^*$, $\mathbf{x}_2^*$, ..., $\mathbf{x}_N^*$) of the original multi-objective problem in (1) have already been given. In this case, our formulations in Section 2 can be viewed as single-objective and multi-objective solution selection problems. Any subset S of the N non-dominated solutions can be represented by a binary string of the length N: $S = s_1 s_2 \ldots s_N$ where $s_j = 1$ and $s_j = 0$ mean the inclusion of the j-th non-dominated solution $\mathbf{x}_j^*$ in the solution set S and its exclusion from S, respectively. We can easily use evolutionary algorithms for solution selection.

4. EXPERIMENTAL RESULTS

We generated a large number of non-dominated solutions of a three-objective 500-item knapsack problem by MOEA/D [6]. Then we used a single-objective genetic algorithm to choose 40 non-dominated solutions that maximize their hypervolume. Fig. 1 shows an example of selected 40 non-dominated solutions. We also used NSGA-II to find solution sets along the tradeoff curve with respect to the number of non-dominated solutions and their hypervolume. Experimental results are shown in Fig.2 where an open circle denotes a set of non-dominated solutions of the three-objective knapsack problem (e.g., 40 solutions in Fig. 1).

5. CONCLUDING REMARKS

We formulated hypervolume maximization as single-objective and multi-objective problems. We also explained our formulations through computational experiments on a three-objective problem.

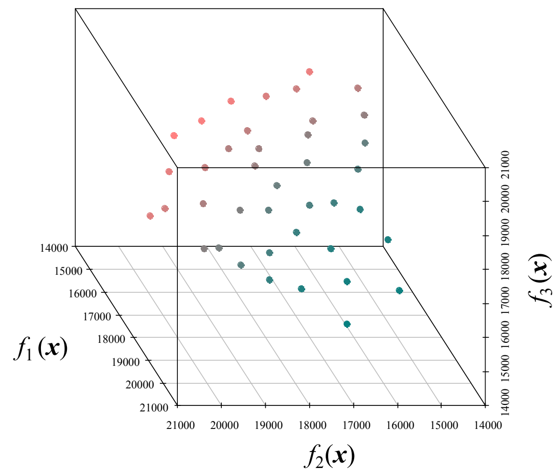

Figure 1. An obtained solution set (single-objective approach).

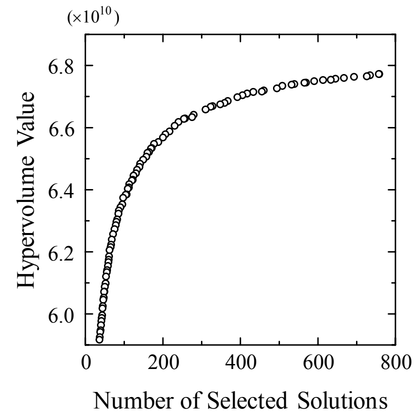

Figure 2. Obtained solution sets (multi-objective approach).

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