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Review of Coevolutionary Developments of Evolutionary Multi-Objective and Many-Objective Algorithms and Test Problems

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***Abstract*—In the evolutionary multi-objective optimization (EMO) community, some well-known test problems have been frequently and repeatedly used to evaluate the performance of EMO algorithms. When a new EMO algorithm is proposed, its performance is evaluated on those test problems. Thus algorithm development can be viewed as being guided by test problems. A number of test problems have already been designed in the literature. Since the difficulty of designed test problems is usually evaluated by existing EMO algorithms through computational experiments, test problem design can be viewed as being guided by EMO algorithms. That is, EMO algorithms and test problems have been developed in a coevolutionary manner. The goal of this paper is to clearly illustrate such a coevolutionary development. We categorize EMO algorithms into four classes: non-elitist, elitist, many-objective, and combinatorial algorithms. In each category of EMO algorithms, we examine the relation between developed EMO algorithms and used test problems. Our examinations of test problems suggest the necessity of strong diversification mechanisms in many-objective EMO algorithms such as SMS-EMOA, MOEA/D and NSGA-III.**

Keywords*—Evolutionary multi-objective optimization (EMO), many-objective optimization, convergence, diversity.

I. Introduction

Evolutionary multi-objective optimization (EMO) has been an active research area in the field of evolutionary computation in the last two decades. About 20 years ago, some well-known classical EMO algorithms were proposed such as MOGA by Fonseca & Fleming in 1993 [1], NPGA by Horn et al. in 1994 [2], and NSGA by Srinivas & Deb in 1995 [3]. They are often categorized as non-elitist EMO algorithms [4] since elite preservation strategies are not used. Common characteristics of those EMO algorithms are Pareto dominance-based fitness evaluation and diversity maintenance. Then, around 2000, high performance EMO algorithms were proposed such as SPEA by Zitzler & Thiele in 1999 [5] and NSGA-II by Deb et al. in 2002 [6]. They are categorized as elitist EMO algorithms [4]. They have elite preservation strategies in addition to Pareto dominance-based fitness evaluation and diversity maintenance. It was shown in Zitzler & Thiele [5] that the search ability of non-elitist classical EMO algorithms was clearly improved by the use of an elite preservation strategy.

Recently it was repeatedly reported in the literature that Pareto dominance-based elitist EMO algorithms did not work well on many-objective test problems [7]. In response to those reports, powerful EMO algorithms were proposed for many-objective optimization around 2010 such as SMS-EMOA by Beume et al. in 2007 [8], HypE by Bader & Zitzler in 2011 [9] and NSGA-III by Deb & Jain in 2014 [10]. Those algorithms are characterized by new fitness evaluation mechanisms for efficient many-objective search. In this paper, we categorize them as many-objective EMO algorithms.

The main research stream in the EMO community has been the non-elitist, elitist and many-objective EMO algorithms. They were proposed and/or used for continuous multi-objective optimization. However, some EMO algorithms were proposed for combinatorial multi-objective optimization such as MOGA by Murata & Ishibuchi in 1995 [11], MOGLS by Ishibuchi & Murata in 1998 [12], and MOGLS by Jaszkiewicz in 2002 [13], [14]. Their main characteristics are problem-specific coding (e.g., permutation coding) and weighed scalarizing function-based fitness evaluation. Local search is often incorporated into those algorithms. While they have elite preservation strategies, they are not categorized as elitist EMO algorithms [4]. In this paper, we categorize them as combinatorial EMO algorithms.

Whereas combinatorial EMO algorithms often show high search ability [14], they had been almost always outside the main research stream of the EMO community until MOEA/D was proposed by Zhang & Li in 2007 [15]. MOEA/D can be viewed as an improved version of a cellular EMO algorithm proposed by Murata et al. in 2001 [16], where a scalarizing function with a different weight vector was assigned to each cell in the same manner as the decomposition mechanism of MOEA/D. MOEA/D has often been used as a basic framework to combine useful mechanisms for multi-objective and many-objective optimization into a single efficient EMO algorithm. For example, the following mechanism can be implemented in the MOEA/D framework: Weighted scalarizing function-based fitness evaluation, local search, similar parent recombination, and preference incorporation. An archive population can be stored outside of the main population as in the original version of MOEA/D [15] in order to maintain a set of well-distributed non-dominated solutions using Pareto dominance relation and a diversity maintenance mechanism.

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MOEA/D shows high performance on combinatorial many-objective problems [17]. In this paper, we categorize MOEA/D to the related two categories: many-objective EMO algorithms and combinatorial EMO algorithms.

In this paper, we examine multi-objective test problems used for performance evaluations of EMO algorithms in each of the above-mentioned four categories. By visually examining the relation between the Pareto front and a randomly generated initial population, we demonstrate the necessity of a strong convergence property and/or a strong diversification property to handle each test problem. Then we discuss the relation between the required properties for each test problem and the characteristics of actually developed EMO algorithms in each category. Such an examination of the relation between the used test problems and the developed EMO algorithms is performed in each of the next four sections (i.e., Sections II-V) for the four categories of EMO algorithms, respectively. In Section VI, we summarize our examinations in the previous four sections. A focus is placed on the necessity of diversification in many-objective EMO algorithms. Finally, we conclude this paper in Section VII where some future research topics are suggested.

II. Test Problems for Non-Elitist EMO Algorithms

As mentioned in Section I, the common characteristics of classical non-elitist EMO algorithms are Pareto dominance-based fitness evaluation and diversity maintenance. No elite preservation strategies are used in those EMO algorithms. No convergence improvement mechanisms are used, either. In this section, we show that these characteristics of non-elitist EMO algorithms are closely related to the properties of some frequently-used test problems in the 1990s.

A. Schaffer's Test Problem (1984)

Schaffer's test problem [18] is the following two-objective minimization problem with a single decision variable x:

$$\text{Minimize} \quad f_1(x) = x^2,\ f_2(x) = (x-2)^2 .$$

This test problem was popular and frequently used in the 1990s such as the NPGA paper [2] and the NSGA paper [3].

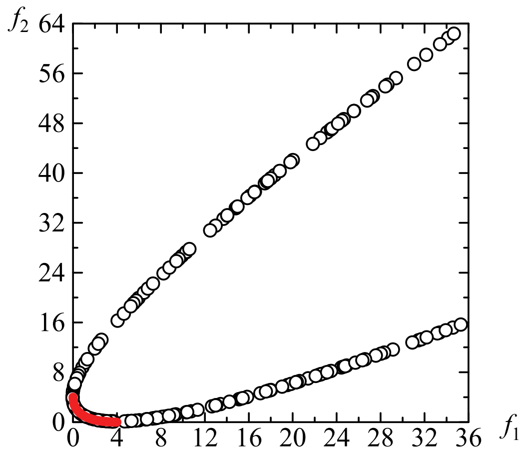

Fig. 1. Randomly generated 200 solutions of the Schaffer's test problem [18] for $-6 \le x \le 6$. The Pareto front is shown by a red curve.

Randomly generated 200 solutions are shown by open circles for the case of $-6 \le x \le 6$ in Fig. 1 where a red curve is the Pareto front. We can see from Fig. 1 that some of the randomly generated solutions are Pareto optimal. This means that the search for Pareto optimal solutions can be performed by choosing non-dominated solutions in the current population. Diversity maintenance is needed to search for a set of well-distributed (i.e., evenly distributed) Pareto optimal solutions on the Pareto front. Since some randomly generated solutions are Pareto optimal, no strong convergence properties are needed. These requirements for the Schaffer's problem are consistent with the characteristics of non-elitist EMO algorithms.

B. F3 of Srinivas & Deb (1995)

The following two-objective minimization problem with two decision variables was called F3 in Srinivas & Deb [3]:

$$\begin{aligned} \text{Minimize} \quad & f_1(x_1,\, x_2) = (x_1-2)^2 + (x_2-1)^2 + 2\,, \\ & f_2(x_1,\, x_2) = 9x_1 - (x_2-1)^2\,, \\ \text{where} \quad & -20 \le x_1 \le 20\,,\ -20 \le x_2 \le 20\,, \\ & x_1^2 + x_2^2 - 225 \le 0,\ \ x_1 - 3x_2 + 10 \le 0\,. \end{aligned}$$

Randomly generated 200 solutions are shown in Fig. 2 together with the Pareto front (a red curve). In Fig. 2, open circles are feasible solutions while gray circles are infeasible solutions. From Fig. 2, we can see that many solutions are infeasible. We can also see that some feasible solutions are on or very close to the Pareto front. It seems from Fig. 2 that no strong convergence properties are needed to search for Pareto optimal solutions of the F3 test problem. So, it is not likely that EMO algorithms with strong convergence properties are developed for the F3 test problem. This discussion is consistent with the characteristics of non-elitist EMO algorithms.

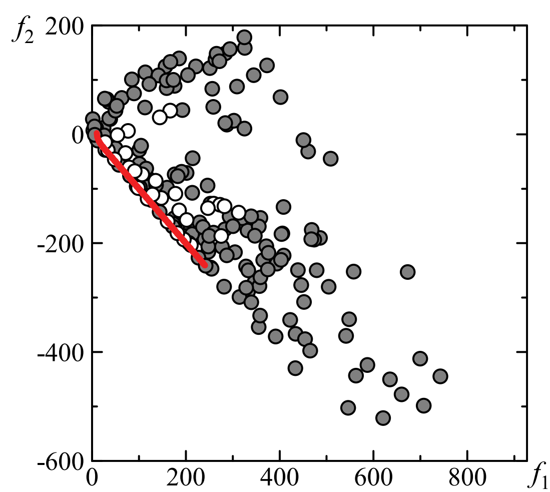

Fig. 2. Randomly generated 200 solutions of the F3 test problem [3]. Feasible and infeasible solutions are shown by open and gray circles, respectively. The Pareto front is shown by a red curve.

C. A Test Problem of Fonseca & Fleming (1995)

The following two-objective minimization problem was shown in a survey paper by Fonseca & Fleming in 1995 [19]:

$$\text{Minimize } f_1(x_1, x_2) = 1 - \exp(-(x_1 - 1)^2 - (x_2 + 1)^2),$$
$$f_2(x_1, x_2) = 1 - \exp(-(x_1 + 1)^2 - (x_2 - 1)^2),$$
$$\text{where } -4 \le x_1 \le 4, \; -4 \le x_2 \le 4.$$

Randomly generated 200 solutions are shown in Fig. 3 together with the Pareto front (a red curve). We can see from Fig. 3 that some solutions are on or very close to the Pareto front. Those solutions can be selected using Pareto dominance-based fitness evaluation. We can also see that those solutions are not uniformly distributed over the entire Pareto front. Thus diversity maintenance is important in Fig. 3 whereas no strong convergence properties are needed. These requirements in Fig. 3 are consistent with the characteristics of non-elitist EMO algorithms.

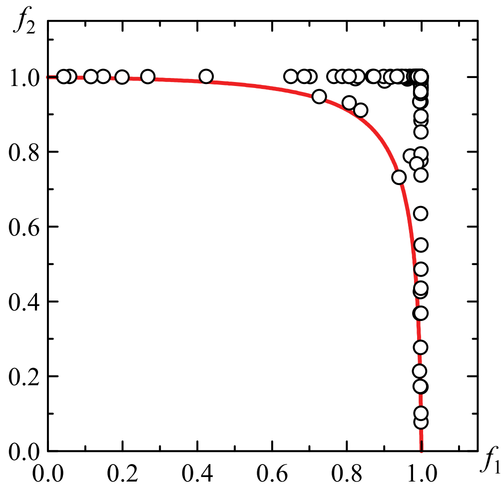

Fig. 3. Randomly generated 200 solutions of the test problem in Fonseca & Fleming [19]. The Pareto front is shown by a red curve.

III. Test Problems for Elitist EMO Algorithms

Elitist EMO algorithms such as SPEA and NSGA-II have elite preservation mechanisms in addition to Pareto dominance-based fitness evaluation and diversity preservation. Thus elitist EMO algorithms have stronger convergence properties than non-elitist algorithms. The corresponding requirements are observed in test problems used for performance evaluation of elitist EMO algorithms as we will show in this section.

A. ZDT Test Suite (2000)

The ZDT test suite was designed by Zitzler et al. in 2000 [20]. In the ZDT test suite, five test problems (ZDT1 - ZDT4 and ZDT6) were used by Deb et al. in 2002 to evaluate the performance of their NSGA-II [6]. Randomly generated 200 solutions and the Pareto front of ZDT1 are shown in Fig. 4. No solutions are close to the Pareto front in Fig. 4, which is totally different from Figs. 1-3 in Section II. This observation suggests the necessity of a strong convergence property to efficiently search for Pareto optimal solutions of ZDT1. This requirement is consistent with the characteristics of elitist EMO algorithms. In other words, ZDT1 is helpful to demonstrate high search ability of EMO algorithms with strong convergence properties.

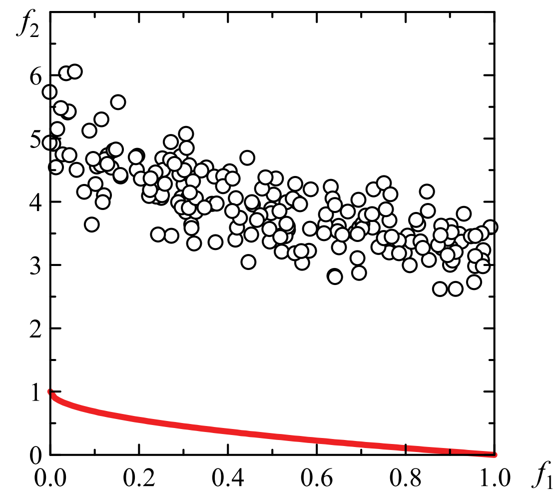

Fig. 4. Randomly generated 200 solutions and the Pareto front (red curve) of the ZDT1 test problem with two decision variables: $0 \le x_1 \le 1$ and $0 \le x_2 \le 1$.

In Fig. 5, we show randomly generated 200 solutions and the Pareto front of ZDT2. As in Fig. 4, no solutions are close to the Pareto front in Fig. 5. This observation suggests the necessity of strong convergence properties of EMO algorithms to efficiently search for Pareto optimal solutions of ZDT2. The same observation is obtained from the other ZDT test problems.

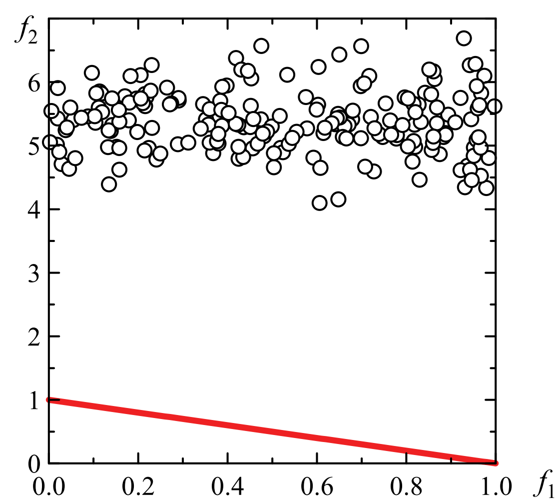

Fig. 5. Randomly generated 200 solutions and the Pareto front (red line) of the ZDT2 test problem with two decision variables: $0 \le x_1 \le 1$ and $0 \le x_2 \le 1$.

From Fig. 4 and Fig. 5, we can also see that the length of the red line is almost the same as the width of the population of solutions. It seems that a set of well-distributed solutions over the entire Pareto front can be obtained by strongly pushing the population towards the Pareto front. No strong diversification mechanism is needed in Fig. 4 and Fig. 5 (since we do not have to increase the width of the population). These observations are consistent with the characteristics of elitist EMO algorithms.

B. Multi-Objective Knapsack Problems (1999)

Zitzler & Thiele used multi-objective knapsack problems with 2-4 objectives and 250, 500 and 750 items to demonstrate high performance of their SPEA algorithm in 1999 [5]. While knapsack problems are combinatorial optimization problems, we categorize SPEA as an elitist EMO algorithm since SPEA (together with NSGA-II) has been handled as a representative of elitist EMO algorithms in the literature. Moreover, SPEA has been frequently used for continuous optimization.

In Fig. 6, we show an initial population of randomly generated 200 solutions (open circles) and the Pareto front (red circles) of the two-objective 500-item knapsack problem. A greedy repair method in Zitzler & Thiele [5] is applied when a randomly generated solution is infeasible. That is, Fig. 6 shows an initial population of 200 solutions after repair.

We can see from Fig. 6 that strong convergence properties are needed since an initial population is far from the Pareto front. This observation is consistent with the use of an elite preservation strategy in SPEA by Zitzler & Thiele [5]. They clearly demonstrated that SPEA outperformed non-elitist EMO algorithms using nine multi-objective knapsack problems including the two-objective 500-item problem in Fig. 6.

We can also see from Fig. 6 that the spread of the Pareto front (the length of the red line) is much larger than the width of the population. This observation suggests the necessity of strong diversification properties. However, SPEA has no strong diversification mechanism except for clustering-based diversity maintenance. As a result, SPEA finds many solutions around the center of the Pareto front and no solutions around its two edges. It seems that this difficulty was not a severe issue when SPEA was proposed in 1999. This is because there was a large gap in performance on multi-objective knapsack problems between SPEA and classical non-elitist EMO algorithms.

Most elitist EMO algorithms such as SPEA and NSGA-II do not have very strong diversification properties. This may be because other test problems such as the ZDT suite do not need strong diversification properties. It is difficult for those elitist EMO algorithms to efficiently find well-distributed solutions over the entire Pareto front in Fig. 6 [14]. Some strong diversification mechanisms have been proposed for those elitist EMO algorithms (e.g., biased mating selection [21]).

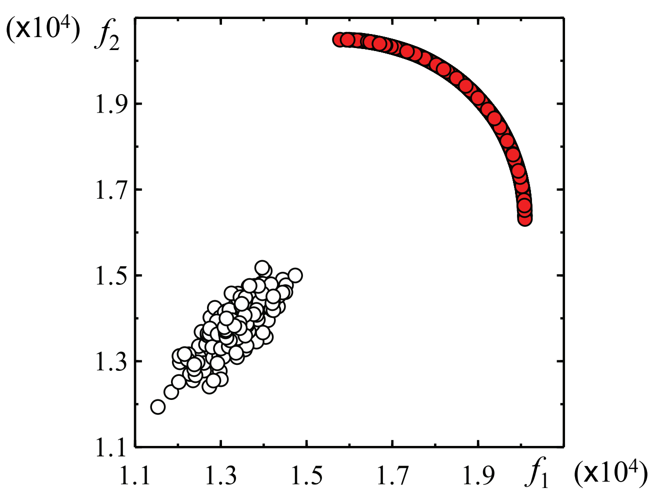

Fig. 6. Randomly generated 200 solutions (open circles) and the Pareto front (red circles) of the two-objective 500-item knapsack problem [5].

IV. Test Problems for Many-Objective EMO Algorithms

Multi-objective problems with four or more objectives are often referred to as many-objective problems [7]. It is well-known that Pareto dominance-based elitist EMO algorithms such as SPEA [5] and NSGA-II [6] do not work well on many-objective problems whereas they show high search ability on multi-objective problems with two or three objectives.

A. DTLZ Test Suite (2002)

The DTLZ test suite was designed by Deb et al. in 2002 [22]. The main feature of this test suite is that the number of objectives can be arbitrarily specified. Recently, the DTLZ test suite has been frequently used for performance evaluation of many-objective EMO algorithms in the literature such as the SMS-EMOA paper [8], the HypE paper [9], the NSGA-III paper [10], and other many-objective EMO papers [23], [24].

Let M be the number of objectives. The number of decision variables (say n) of an M-objective DTLZ problem is specified as $n = M + k - 1$ where k is a parameter. In this paper, we specify the value of k as $k = 5$. In Fig. 7 and Fig. 8, we show randomly generated 200 solutions and the Pareto front of a two-objective DTLZ1 problem ($M = 2$, $n = 6$). Fig. 8 shows the bottom-left corner of Fig. 7.

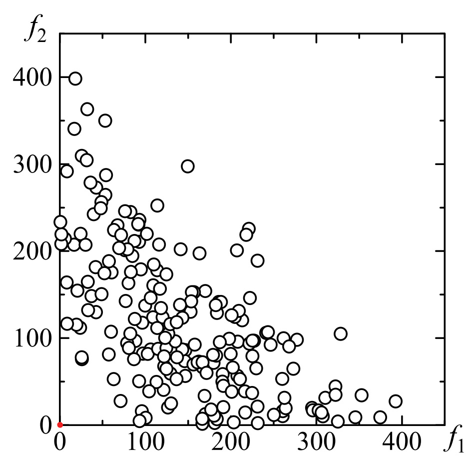

Fig. 7. Randomly generated 200 solutions (open circles) and the Pareto front (red) of the two-objective DTLZ1 problem [22].

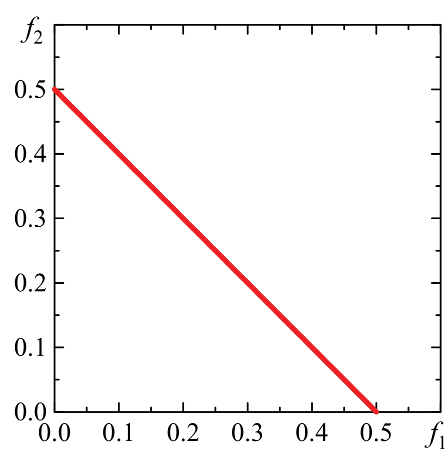

Fig. 8. A part of Fig. 7 including the Pareto front (red) of DTLZ1 [22].

From Fig. 7 and Fig. 8, we can see that no solutions are close to the Pareto front. This observation suggests the necessity of strong convergence properties. We can also see that the diversity of randomly generated solutions in the objective space is much larger than the spread of the Pareto front. This observation suggests that no strong diversification properties are needed in Fig. 7 and Fig. 8.

Fig. 9 shows randomly generated 200 solutions of a ten-objective DTLZ1 problem (M = 10, n = 14). The generated solutions in the ten-dimensional objective space are projected to its two-dimensional subspaces in Fig. 9. The projection to the f_9 - f_{10} subspace around the bottom-right corner is similar to Fig. 7. In Fig. 9, the diversity of the projected solutions becomes smaller from the bottom-right corner to the top-left corner. For example, the diversity of the projected solutions in the f_1 - f_2 subspace around the top-left corner of Fig. 9 is much smaller than that in the f_9 - f_{10} subspace around the bottom-right corner. However, the projected solutions even in the f_1 - f_2 subspace still have much larger diversity than the Pareto front of the ten-objective DTLZ1 problem as shown in Fig. 10. Fig. 10 is an enlarged version of the projection to the f_1 - f_2 subspace (i.e., an enlarged version of the plot at the second row and the first column of Fig. 9). The Pareto front in the ten-dimensional objective space of DTLZ1 is also projected to the f_1 - f_2 subspace in Fig. 10.

Figs. 7-10 show that many-objective EMO algorithms for DTLZ1 need strong convergence properties but no strong diversification properties. These observations are consistent with the characteristics of SMS-EMOA [8], HypE [9] and NSGA-III [10]. The lack of strong diversification properties in those EMO algorithms can be confirmed by their applications to the two-objective knapsack problem in Fig. 6 (see also [17]).

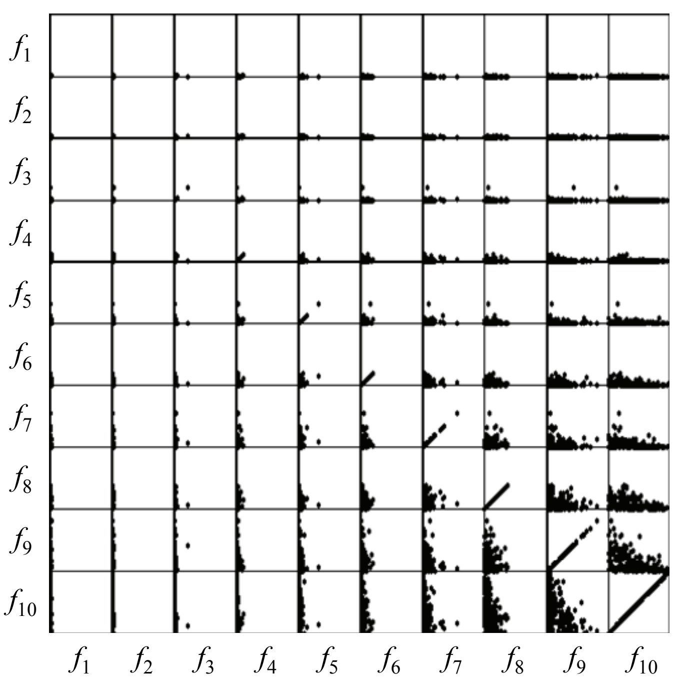

Fig. 9. Projections of randomly generated 200 solutions of the ten-objective DTLZ1 problem [22] into two-dimensional subspaces. The f_i-f_j subspace is shown at the jth row and the ith column.

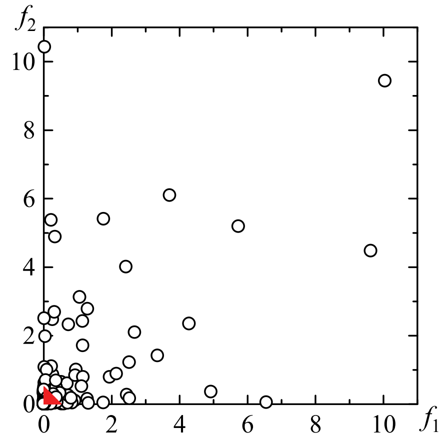

Fig. 10. Projection to the f_1 - f_2 subspace (10-objective DTLZ1 [22]).

B. *WFG Test Suite (2006)*

The WFG test suite with a variety of many-objective test problems was proposed by Huband et al. in 2006 [25]. This test suite was also frequently used for performance evaluation of many-objective EMO algorithms in the literature such as the HypE paper [9], the NSGA-III paper [10], and other many-objective EMO papers (e.g., [23], [24]). Due to the page limitation as the conference paper, detailed explanations on the WFG test suite are omitted. Each test problem in the WFG test suite has different features. Whereas almost all test problems seem to require strong convergence properties and moderate diversity maintenance properties, totally different observations are obtained from WFG1. In Fig. 11, the diversity of randomly generated solutions (black) is much smaller than the spread of the Pareto front (red) of a three-objective WFG1 problem. Fig. 11 suggests the necessity of strong diversification properties to efficiently search for Pareto optimal solutions over the entire Pareto front of WFG1. However, we do not obtain similar observations from the other test problems in the WFG test suite.

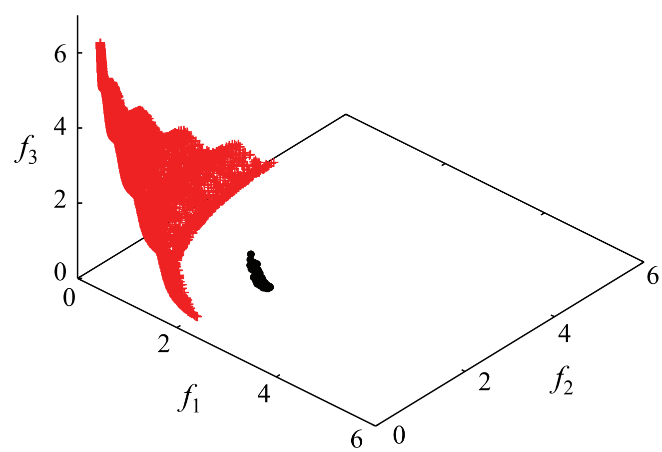

Fig. 11. Randomly generated 200 solutions (black) and the Pareto front (red) of a three-objective WFG1 problem [25] with k =2 and l = 4.

C. *Many-Objective Knapsack Problems (1999, 2007)*

Multi-objective knapsack problems in Zitzler & Thiele [5] have 2-4 objectives and 250, 500 and 750 items. The four-objective knapsack problems can be viewed as many-objective test problems whereas they were used as multi-objective test problems in 1999 [5]. Multi-objective knapsack problems are scalable test problems with large flexibility. This is because we can arbitrarily specify the number of objectives, the number of decision variables, and the number of constraint conditions.

Many-objective knapsack problems with more than four objectives have been used in some studies on many-objective optimizations such as the HypE paper [9] and some others (e.g., [17]). One of the earliest well-known studies with the use of many-objective knapsack problems for performance evaluation of many-objective EMO algorithm is Sato et al. in 2007 [26]. Knapsack problems with 2-10 objectives were used to show strong effects of their control mechanism of the dominance area on the performance of elitist EMO algorithms.

V. Test Problems for Combinatorial EMO Algorithms

In general, combinatorial multi-objective problems have totally different features from continuous ones. They usually need problem-specific coding and genetic operators. As a result, combinatorial EMO algorithms often have totally different characteristics from continuous EMO algorithms. In this section, we examine the relation between combinatorial EMO algorithms and combinatorial multi-objective test problems.

A. Multi-Objective Flowshop Scheduling (1995)

Multi-objective flowshop scheduling problems were used by Murata & Ishibuchi in 1995 [11] to examine their random weight EMO algorithm (RW-MOGA [4]). In Fig. 12, we show randomly generated 200 solutions (open circles) and the Pareto front (red circles) of a two-objective flowshop scheduling problem with five machines and ten jobs.

It should be noted that a very small test problem is used in Fig. 12 to find its exact Pareto front (to examine all possible permutations of jobs). While such a small-size test problem is used, no randomly generated solutions are very close to the Pareto front. This observation is clearly different from Figs. 1-3 of the continuous multi-objective test problems in the mid-1990s where some solutions are very close to the Pareto front.

In Fig. 12, we can see that strong convergence properties are needed to efficiently search for Pareto optimal solutions. Actually, RW-MOGA in 1995 [11] has an elite preserving mechanism to achieve a strong convergence properties. The necessity of further stronger convergence properties for large-scale flowshop scheduling problems led to the incorporation of local search into RW-MOGA (i.e., a proposal of MOGLS by Ishibuchi & Murata in 1996 [27]). In Fig. 12, it seems that no strong diversification properties are needed. This observation is consistent with the characteristics of RW-MOGA and MOGLS. They have strong convergence properties (i.e., elitist EMO and local search) but no explicit diversification mechanisms.

B. Multi-Objective TSP (2002)

Jaszkiewicz used two-objective and three-objective TSP test problems with 50 and 100 cities in 2002 [13] for performance evaluation of his MOGLS algorithm. Fig. 13 shows randomly generated 200 solutions (open circles) and the Pareto front (red circles) of a two-objective 100-city problem (kroAB100). We can see from Fig. 13 that a population of randomly generated initial solutions is far from the Pareto front. This observation suggests the need of very strong convergence properties. We can also see that the diversity of the generated population is much smaller than the size of the Pareto front (i.e., the length of the red line in Fig. 3). This observation suggests the need of very strong diversification properties.

Jaszkiewicz's MOGLS in 2002 [13] can be viewed as an improved version of the original MOGLS in 1996 [27]. The main improvement is the use of scalarizing function-based strong selection pressure to choose a pair of good and similar parents for recombination (i.e., to recombine a pair of good solutions with high similarity). Strong selection pressure can also improve the performance of the original MOGLS.

Good parent selection improves the convergence property while similar parent selection improves the diversification property [21]. That is, the improvement was performed for convergence and diversity. This is consistent with difficulties in Fig. 13 in comparison with Fig. 12: All solutions in Fig. 13 are far from the Pareto front, and the diversity of solutions in Fig. 13 is much smaller than the size of the Pareto front.

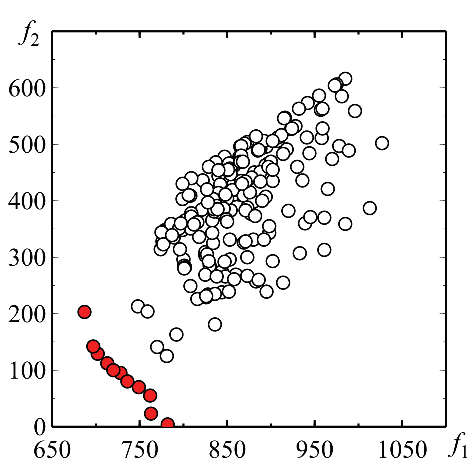

Fig. 12. Randomly generated 200 solutions (open circles) and the Pareto front (red circles) of the five-machine ten-job flowshop scheduling problem [11].

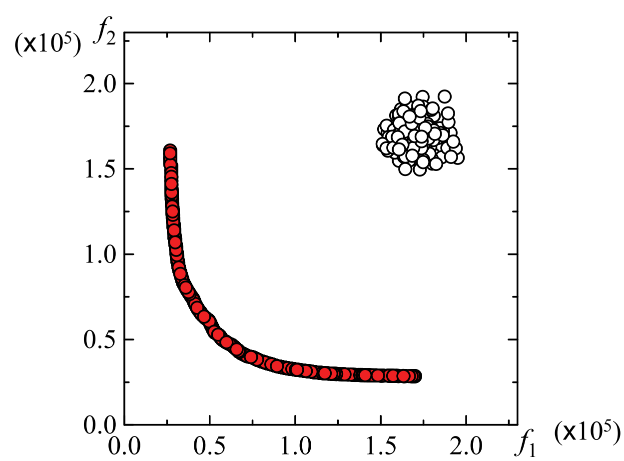

Fig. 13. Randomly generated 200 solutions (open circles) and the Pareto front (red circles) of the two-objective 100-city TSP test problem in [13].

C. Multi-Objective Knapsack Problems (2002, 2007)

Multi-objective knapsack problems in Zitzler & Thiele [5] shown in Fig. 6 were also used by Jaszkiewicz to demonstrate high performance of his MOGLS algorithm in 2002 [14]. Since strong convergence and diversification properties are needed to search for Pareto optimal solutions over the entire Pareto front in Fig. 6, Jaszkiewicz's MOGLS outperforms the original MOGLS and some elitist EMO algorithms.

Multi-objective knapsack problems in Zitzler & Thiele [5] were also used by Zhang & Li to examine the search ability of their MOEA/D algorithm in 2007 [15]. Since MOEA/D has strong convergence and diversification properties, good results were reported on multi-objective knapsack problems in [15]. In MOEA/D, a single multi-objective problem is decomposed into multiple single-objective problems. A scalarizing function with a different weight vector is assigned to each single-objective problem as its objective (fitness) function.

MOEA/D has a strong diversification property since a number of uniformly sampled weight vectors are used to generate single-objective problems. Each single-objective problem has a different weight vector. The number of the single-objective problems is the same as the number of the weight vectors. For each weight vector, a pre-specified number of the nearest weight vectors are defined as its neighbors. The same neighborhood structure is applied to the corresponding single-objective problems. Each single-objective problem is solved in a cooperative manner with its neighboring problems. That is, a new solution for each single-objective problem is generated by genetic operations from its neighboring problems. Such a local use of genetic operations has the same effect of similar parent recombination (diversity maintenance). The generated solution is compared with the current solutions in the neighborhood using the weight vector of each neighbor. If a very good solution is generated, the current solutions of multiple neighbors would be replaced with the generated one. Such a local replacement mechanism helps to speed-up the convergence of solutions towards the Pareto front.

Strong convergence and diversification properties of MOEA/D are consistent with the required properties to efficiently search for Pareto optimal solutions over the entire Pareto front of the knapsack problem in Fig. 6.

VI. Discussions and Further Examinations

In this paper, we have already explained that there exist strong relations between developed EMO algorithms and test problems used for their performance evaluation. For example, strong convergence properties are not needed to search for Pareto optimal solutions of simple test problems in the mid-1990s. In response to this feature of those test problems, well-known classical non-elitist EMO algorithms such as MOGA [1], NPGA [2] and NSGA [3] do not have strong convergence properties. Strong convergence properties are needed to efficiently search for Pareto optimal solutions of multi-objective problems in the ZDT suite in 2000 [20]. Strong convergence properties are also needed to handle multi-objective knapsack problems in 1999 [5]. In response to this feature of those test problems, elitist EMO algorithms have strong convergence properties (SPEA [5] and NSGA-II [6]).

Among frequently-used combinatorial multi-objective test problems, multi-objective flowshop scheduling problems in 1998 [12] need strong convergence properties. Local search was introduced in MOGLS of Ishibuchi & Murata [12] to push the population towards the Pareto front. Multi-objective TSP problems in 2002 [13] need both strong convergence and strong diversification. In response to these requirements, a very strong selection pressure was introduced to mating selection of MOGLS of Jaszkiewicz [13] in order to choose a pair of good and similar parents using a scalarizing function with a random weight vector. Similar parent recombination has a positive effect on diversity maintenance.

The DTLZ test suite in 2002 [22] and the WFG test suite in 2006 [25] have been frequently and repeatedly used in the field of evolutionary many-objective optimization. Whereas very strong convergence properties are needed to handle many-objective test problems in the DTLZ and WFG test suites, strong diversification properties are not always needed. This may suggest the possibility that existing many-objective EMO algorithms do not work well on test problems which need strong diversification properties.

In Fig. 14, we show experimental results of a single run of SMS-EMOA and MOEA/D with the weighted Tchebycheff function under the same settings as in our previous study [17]. As shown in Fig. 14, a wide variety of solutions are not obtained by SMS-EMOA.

In computational experiments of MOEA/D in Fig. 14, we specified the reference point $\boldsymbol{z}^* = (z_1^*, z_2^*)$ for the weighted Tchebycheff function at the tth generation of MOEA/D as

$$z_i^* = \alpha \cdot \max\{f_i(\boldsymbol{x}) \mid \boldsymbol{x} \in \Omega(1) \cup \Omega(2) \cup \cdots \cup \Omega(t)\}, \; i = 1, 2,$$

where $\Omega(t)$ is the population at the tth generation. Whereas α = 1.0 is the standard setting for the parameter α, it is specified as α = 1.1 in Fig. 14. This specification is to strengthen the diversification property of MOEA/D (actually, α = 1.1 was also used in Zhang & Li [15] only for multi-objective knapsack problems). Fig. 15 shows experimental results of MOEA/D for two specifications: α = 1.0 and α = 1.2. We can see that a large diversity of solutions is not obtained from α = 1.0.

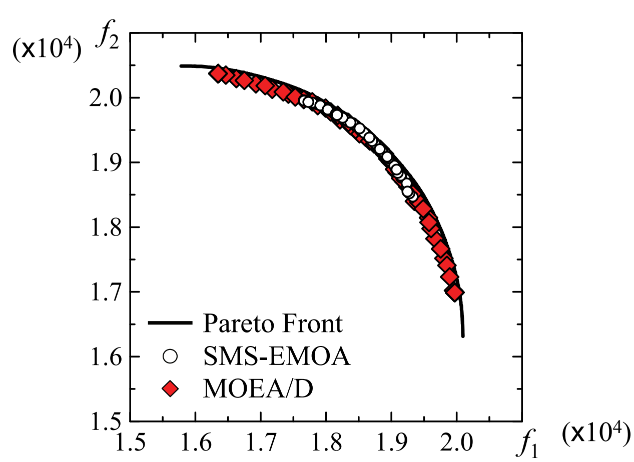

Fig. 14. Results of a single run of SMS-EMOA and MOEA/D on the two-objective 500-item knapsack problem.

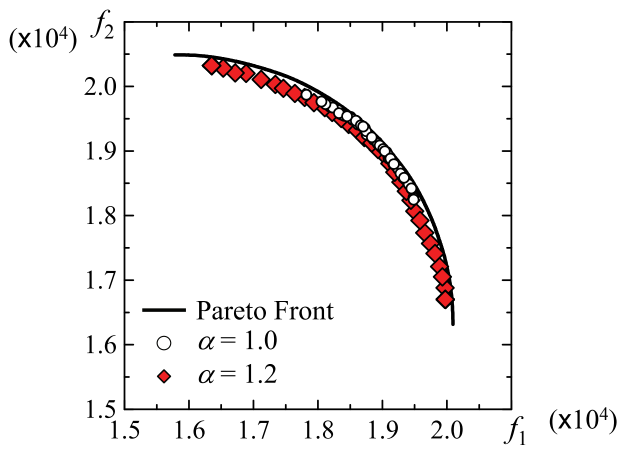

Fig. 15. Results of MOEA/D with different specifications of the reference point in the weighted Tchebycheff function.

VII. Concluding Remarks

In this paper, we examined the relation between developed algorithms and used test problems in the EMO community. Clear relations between them were observed: EMO algorithms and test problems have been developed in a coevolutionary manner. This observation suggests that the design of new test problems will drive the development of new EMO algorithms.

Future research includes further examination of existing multi-objective and many-objective test problems from a point of view of coevolutionary developments of EMO algorithms and test problems. It is an interesting future research topic to design difficult test problems for existing EMO algorithms, which will stimulate novel EMO algorithm developments. For example, it may be a promising research direction to design many-objective test problems that need strong diversification as well as strong convergence. Another important issue to be discussed is the design of test problems that are similar to real-world multi-objective problems, which may stimulate more practically-useful EMO algorithm developments.

References

[1] C. M. Fonseca and P. J. Fleming, "Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization," *Proc. of the 5th International Conference on Genetic Algorithms*, pp. 416-423, 1993.

[2] J. Horn, N. Nafpliotis, and D. E. Goldberg, "A niched Pareto genetic algorithm for multi-objective optimization," *Proc. of the 1st IEEE International Conference on Evolutionary Computation*, pp. 82-87, 1994.

[3] N. Srinivas and K. Deb, "Muiltiobjective optimization using nondominated sorting in genetic algorithms," *Evolutionary Computation*, vol. 2, no. 3, pp. 221-248, 1995.

[4] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, John Wiley & Sons, Chichester, 2001.

[5] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach," *IEEE Trans. on Evolutionary Computation*, vol. 3, no. 4, pp. 257-271, 1999.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation*, vol. 6, no. 2, pp. 182-197, 2002.

[7] H. Ishibuchi, N. Tsukamoto, and Y. Nojima, "Evolutionary many-objective optimization: A short review," *Proc. of 2008 IEEE Congress on Evolutionary Computation*, pp. 2424-2431, Hong Kong, China, June 1-6, 2008.

[8] N. Beume, B. Naujoks, and M. Emmerich, "SMS-EMOA: Multiobjective selection based on dominated hypervolume," *European Journal of Operational Research*, vol. 180, no. 3, pp. 1653-1669, 2007.

[9] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," *Evolutionary Computation*, vol. 19 no. 1, pp. 45-76, 2011.

[10] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, Part I: Solving problems with box constraints," *IEEE Trans. on Evolutionary Computation*, vol. 18, no. 4, pp. 577-601, August 2014.

[11] T. Murata and H. Ishibuchi, "MOGA: Multi-objective genetic algorithms," *Proc. of the 2nd IEEE International Conference on Evolutionary Computation*, pp. 289-294, 1995.

[12] H. Ishibuchi and T. Murata, "A multi-objective genetic local search algorithm and its application to flowshop scheduling," *IEEE Trans. on Systems, Man, and Cybernetics - Part C: Applications and Reviews*, vol. 28, no. 3, pp. 392-403, 1998.

[13] A. Jaszkiewicz, "Genetic local search for multi-objective combinatorial optimization," *European Journal of Operational Research*, vol. 137, no. 1, pp. 50-71, 2002.

[14] A. Jaszkiewicz, "On the performance of multiple-objective genetic local search on the 0/1 knapsack problem - A comparative experiment," *IEEE Trans. on Evolutionary Computation*, vol. 6, no. 4, pp. 402-412, 2002.

[15] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. on Evolutionary Computation*, vol. 11, no. 6, pp. 712-731, 2007.

[16] T. Murata, H. Ishibuchi, and M. Gen, "Specification of genetic search directions in cellular multi-objective genetic algorithm," *Lecture Notes in Computer Science 1993: EMO 2001*, pp. 82-95, Springer, 2001.

[17] H. Ishibuchi, N. Akedo, and Y. Nojima, "Behavior of multi-objective evolutionary algorithms on many-objective knapsack problems," *IEEE Trans. on Evolutionary Computation* (in press).

[18] J. D. Schaffer, *Some experiments in machine learning using vector evaluated genetic algorithms*, Ph.D. Dissertation, University of Michigan, 1984.

[19] C. M. Fonseca and P. J. Fleming, "An overview of evolutionary algorithms in multiobjective optimization," *Evolutionary Computation*, vol. 3, no. 1, pp. 1-16, 1995.

[20] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173-195, 2000.

[21] H. Ishibuchi, K. Narukawa, N. Tsukamoto, and Y. Nojima, "An empirical study on similarity-based mating for evolutionary multiobjective combinatorial optimization," *European Journal of Operational Research*, vol. 188, no. 1, pp. 57-75, 2008.

[22] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multiobjective optimization test problems," *Proc. of IEEE Congress on Evolutionary Computation*, pp. 825-830, 2002.

[23] Z. He, G. G. Yen, and J. Zhang, "Fuzzy-based Pareto optimality for many-objective evolutionary algorithms," *IEEE Trans. on Evolutionary Computation*, vol. 18, no. 2, pp. 269-285, April 2014.

[24] M. Li, S. Yang, and X. Liu, "Shift-based density estimation for Pareto-based algorithms in many-objective optimization," *IEEE Trans. on Evolutionary Computation*, vol. 18, no. 3, pp. 348-365, June 2014.

[25] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Trans. on Evolutionary Computation*, vo. 10, no. 5, pp. 477-506, 2006.

[26] H. Sato, H. E. Aguirre, and K. Tanaka, "Controlling dominance area of solutions and its impact on the performance of MOEAs," *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, pp. 5-20, Springer, 2007.

[27] H. Ishibuchi and T. Murata, "Multi-objective genetic local search algorithm," *Proc. of the 3rd IEEE International Conference on Evolutionary Computation*, pp. 119-124, Nagoya, 1996.