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Abstract - We discuss the evolution of strategies in a spatial iterated prisoner's dilemma (IPD) game in which each player is located in a cell of a two-dimensional grid-world. Following the concept of structured demes, two neighborhood structures are used. One is for the interaction among players through the IPD game. A player in each cell plays against its neighbors defined by this neighborhood structure. The other is for mating strategies by genetic operations. A new strategy for a player is generated by genetic operations from a pair of parent strings, which are selected from its neighboring cells defined by the second neighborhood structure. After examining the effect of the two neighborhood structures on the evolution of cooperative behavior with standard pairing in game-playing, we introduce a random pairing scheme in which each player plays against a different randomly chosen neighbor at every round (i.e., every iteration) of the game. Through computer simulations, we demonstrate that small neighborhood structures facilitate the evolution of cooperative behavior under random pairing in game-playing.

Index Terms - Game strategies, evolutionary games, iterated prisoner's dilemma, structured demes, random pairing.

I. Introduction

The evolution of cooperative behavior in the iterated prisoner's dilemma (IPD) game has been discussed in many studies (for example, see Axelrod [1], Lindgren [2], and Fogel [3]). In those studies, each player plays against all other players in the current population. A player's strategy, which may be represented as a binary string or a finite state machine, is evolved by operations such as selection, crossover, and/or mutation. The fitness of a player is defined as its average payoff obtained through the IPD game. Some techniques and concepts have been introduced to the IPD game such as the speciation of strategies in Darwen and Yao [4], a two-level evolution model in Vega-Redondo [5], individual recognition in Crowley et al. [6], rule hierarchies in Crowley [7], partner selection in

Ashlock et al. [8], and Q -learning in Sandholm and Crites [9]. The IPD game has also been extended to various cases such as a multi-player version [10], [11] and a spatial version [12]-[15]. In the latter version, each player is fixed spatially in a cell of a grid-world and plays against only its neighboring players. In [12]-[14], basically two extreme strategies (i.e., “always defect” and “always cooperate”) were considered in the spatial IPD game. Grim [15] and Brauchli et al. [16] examined the evolution of stochastic strategies in the spatial IPD game, in which the probability of cooperation depends on the result of the previous round of the IPD game. For a broader review of past work on the IPD game, see Dugatkin [17].

In this paper, we discuss the evolution of strategies in the spatial IPD game with two neighborhood structures: one is for the interaction among players through the IPD game and the other is for mating strategies. Each player in a cell plays against its neighbors defined by the first neighborhood structure. A new strategy for each player is generated by genetic operations performed on a pair of parent strings, which are selected from its neighboring cells defined by the second neighborhood structure. This spatial IPD game can be described in the framework of structured demes [18]-[21]. (The standard non-spatial IPD game is an example of unstructured demes because there is no structure of players.)

A group of individuals within which interactions occur is referred to as a trait group [18]. The trait groups in our spatial IPD game overlap with each other, as in the cases of territorial animals and most plants in which each individual forms the center of its own trait group. In addition to the neighborhood structure for the interaction through the IPD game (i.e., trait groups), we use another neighborhood structure for mating strategies.

The second neighborhood structure for mating is usually much larger than the first one for interaction, as is true in many cases of structured demes such as territorial animals and most plants. For example, neighboring plants fight with each other for water and sunlight in the first neighborhood structure, which is much smaller than the second neighborhood structure where they can disperse their pollen. For details of structured demes and trait groups, see [18]-[21].

The use of two neighborhood structures was examined by Ifti et al. [22] for a continuous prisoner’s dilemma (CPD) model in which the degree of cooperation is represented by the amount of investment (i.e., a non-negative real number). They used an interaction neighborhood for pairing in game-playing and a learning neighborhood for comparing each player’s payoff with those of its neighbors. The fitness of each player was calculated from game-playing against neighbors in the interaction neighborhood. Then each player adopted the amount of investment of the neighbor (i.e., the neighbor’s strategy) with the highest fitness in the learning neighborhood.

In almost all published studies on the IPD game, each player plays against the same opponent for a prespecified number of rounds. In addition to this standard pairing scheme, we also examine the evolution of cooperative behavior in our spatial IPD game using a random pairing scheme in which each player plays against a different randomly chosen neighbor at every round (i.e., every iteration) of the game. In this paper, we first examine the effect of the two neighborhood structures on the

evolution of cooperative behavior using the standard pairing scheme. Simulation results show that cooperative behavior is more easily evolved in the spatial IPD game than the non-spatial one. This result coincides with Wilson’s discussion [18]-[20] on altruistic behavior of individuals in structured and unstructured demes. Next we introduce a random pairing scheme in order to examine the evolution of cooperative behavior in a more difficult situation. In this pairing scheme, the interaction sequence length against the same opponent is minimum because the opponent of each player is selected randomly from its neighbors at every round of the game. It has already been shown that the evolution of cooperative behavior is difficult when the interaction sequence length is short [6], [7], [23]. This is because cooperation in the IPD game is based on reciprocal altruism [24]. Reciprocal altruism is not fostered in the case of a short interaction sequence against the same opponent. Our computer simulations demonstrate that cooperative behavior can be evolved even in such a difficult case when two neighborhood structures are specified appropriately. The main contribution of this paper is twofold. One is the formulation of the spatial IPD game with the two neighborhood structures using the concept of structured demes. The other is the demonstration of the evolution of cooperative behavior in the case of random pairing (i.e., a short interaction sequence against the same opponent).

This paper is organized as follows. Section II describes our spatial IPD game with the two neighborhood structures. Section III examines the effect of the sizes of these neighborhood structures on the evolution of cooperative behavior. Section IV introduces the random pairing scheme and examines the evolution of cooperative behavior. Finally, Section V concludes this paper.

II. Spatial IPD Game with Structured Demes

A. Game Playing in the First Neighborhood Structure

We use a typical payoff matrix in Table 1. When both players cooperate, each receives three points. When both players defect, each player’s payoff is one point. The highest payoff of five is obtained by defecting when the opponent cooperates. In this case, the opponent receives the lowest payoff, zero. A player’s strategy is denoted by a binary string (say s_i , where s_i is the strategy of Player i). The strategy determines the next action based on a finite history of previous plays of the IPD game. We show an example of such a strategy in Table 2, which illustrates how the binary string “10011” called “tit-for-tat” determines its next action based on the previous single round of the game. A player with this strategy cooperates first and then cooperates at the t -th round ($t \geq 2$) only when the opponent cooperated in $(t-1)$ -th round. Every single-round-memory strategy is denoted by a binary string of length five in the same manner as in Table 2.

Table 1. Payoff matrix.

Player's Move	Opponent's Move	
	C: Cooperate	D: Defect
C: Cooperate	Player: 3 Opponent: 3	Player: 0 Opponent: 5
D: Defect	Player: 5 Opponent: 0	Player: 1 Opponent: 1

Table 2. Illustration of the coding of the tit-for-tat strategy "10011".

Player's first move: Cooperate			1
Moves on the preceding round		Suggested move	
Player	Opponent		
D: Defect	D: Defect	D: Defect	0
C: Cooperate	D: Defect	D: Defect	0
D: Defect	C: Cooperate	C: Cooperate	1
C: Cooperate	C: Cooperate	C: Cooperate	1

Here, we assume that each player is located in a cell of a two-dimensional grid-world with the torus structure. We use a 31×31 grid-world. In the 31×31 grid-world, each player plays the IPD game against only its neighbors defined by a neighborhood structure for interaction. Let $N_{IPD}(i)$ be the set of Player i and its neighbors. Player i plays the IPD game against only players in $N_{IPD}(i)$. We examine several specifications of $N_{IPD}(i)$ in computer simulations. Some examples are shown in Fig. 1. The standard non-spatial IPD game can be viewed as the case where $N_{IPD}(i)$ is the same as the entire grid-world. The game is iterated between a player and its neighbor for a prespecified number of rounds (e.g., 100 rounds). After a player completes the IPD game against a prespecified number of its neighbors, the fitness value of the player is calculated as the average payoff obtained from each round of the game. When $N_{IPD}(i)$ for interaction is small, the fitness value of each player is calculated after the IPD game is completed against all of its neighbors. On the other hand, when $N_{IPD}(i)$ is large, a fixed number of opponents is selected randomly for each player from its neighbors. Here, we randomly select five opponents from $N_{IPD}(i)$ to calculate the fitness value of Player i at every generation when $N_{IPD}(i)$ includes more than five neighbors. This setting prevents a combinatorial increase in CPU time with the size of $N_{IPD}(i)$. In computer simulations, we also examine the case where each player always plays against all neighbors in $N_{IPD}(i)$ for comparison.

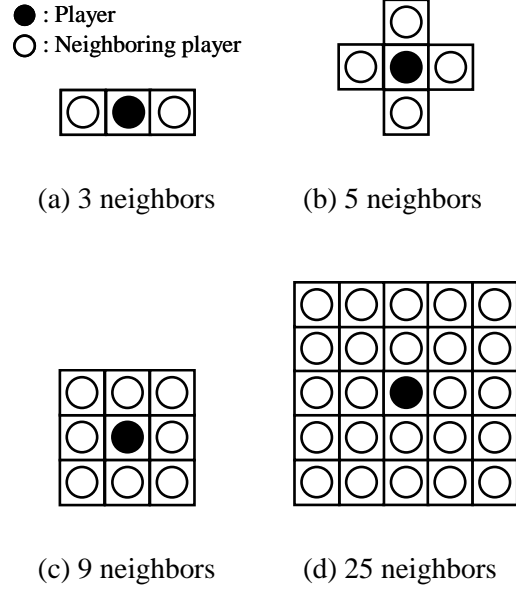


Fig. 1. Examples of neighborhood structures.

B. Evolution in the Second Neighborhood Structure

The evolution of strategies as well as the interaction among players is performed in the two-dimensional grid-world in the framework of cellular genetic algorithms [25]-[27]. Players' strategies are denoted by binary strings of length five as described previously. Since a single player with a single strategy exists in every cell of the two-dimensional grid-world, the population size is the same as the grid-world size 961 (31×31).

A new strategy of a player is generated by selecting two strings from its neighborhood. Let $N_{GA}(i)$ be the set of Player i and its neighbors. A pair of parent strategies is selected from $N_{GA}(i)$ to generate a new strategy for Player i . It should be noted that $N_{GA}(i)$ for mating is not always the same as $N_{IPD}(i)$ for interaction (i.e., $N_{GA}(i) = N_{IPD}(i)$ does not always hold). We examine various specifications of these two neighborhood structures.

Let $f(s_i)$ be the fitness value of Player i with strategy s_i . When a new strategy is to be created, a pair of parent strategies is selected from $N_{GA}(i)$ following roulette wheel selection with a linear scaling:

$$p_i(s_j) = \frac{f(s_j) - f_{\min}(N_{GA}(i))}{\sum_{k \in N_{GA}(i)} \{f(s_k) - f_{\min}(N_{GA}(i))\}} \quad \text{for } j \in N_{GA}(i), \quad (1)$$

where Player j is a neighbor of Player i (or Player i itself), $p_i(s_j)$ is the selection probability of s_j for generating a new strategy of Player i , and $f_{\min}(N_{GA}(i))$ is the minimum fitness value among the

players in $N_{GA}(i)$:

$$f_{\min}(N_{GA}(i)) = \min\{f(s_j) : j \in N_{GA}(i)\}. \quad (2)$$

After selecting two strings using (1), a new string is generated by one-point crossover (Fig. 2) and bit-flip mutation (Fig. 3).

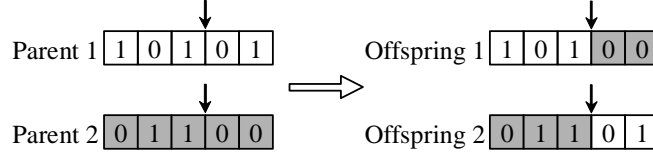


Fig. 2. One-point crossover.

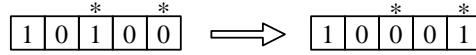


Fig. 3. Bit-flip mutation.

After new strategies for all players are generated by the genetic operations, the current population of strategies is updated by the newly generated strategies. The same procedures (i.e., the IPD game and the genetic operations) are applied to the new population again. In this manner, the generation update is iterated from a randomly generated initial population until a stopping condition is satisfied (e.g., 1000 generations).

III. Computer Simulations with Standard Pairing

Using various specifications of the two neighborhood structures (i.e., $N_{IPD}(i)$ and $N_{GA}(i)$, see Fig. 1), we examined the effect of their specifications on the evolution of cooperative behavior among 961 spatially fixed players in the two-dimensional 31×31 grid-world. We examined all the 36 combinations of the following specifications of the two neighborhood structures:

The number of players in $N_{IPD}(i)$: 3, 5, 9, 25, 49, 961,

The number of players in $N_{GA}(i)$: 3, 5, 9, 25, 49, 961.

The spatial IPD game with 961 players in $N_{IPD}(i)$ and $N_{GA}(i)$ is the same as the standard non-spatial IPD game. When we use the neighborhood structure with three players in Fig. 1 (a) for $N_{GA}(i)$,

strategies of 31 players in each row are never crossed over with strategies of other players in different rows in the two-dimensional 31×31 grid-world. This means that players' strategies cannot be propagated to other players in different rows. In order to avoid this undesirable situation, we use not only the horizontal neighborhood structure in Fig. 1 (a) but also a vertical neighborhood structure for $N_{GA}(i)$ as shown in Fig. 4. For example, $N_{GA}(M)$ in Fig. 4 is horizontal (i.e., $N_{GA}(M) = \{L, M, N\}$) while $N_{GA}(N)$ is vertical (i.e., $N_{GA}(N) = \{I, N, S\}$). More specifically, we specified $N_{GA}(i)$ according to the location of Player i . Let (i_1, i_2) be the location of Player i in the two-dimensional 31×31 grid-world (i.e., $(i_1, i_2) = (1, 1), (1, 2), \dots, (31, 31)$). $N_{GA}(i)$ is horizontal when $(i_1 + i_2)$ is even, otherwise $N_{GA}(i)$ is vertical.

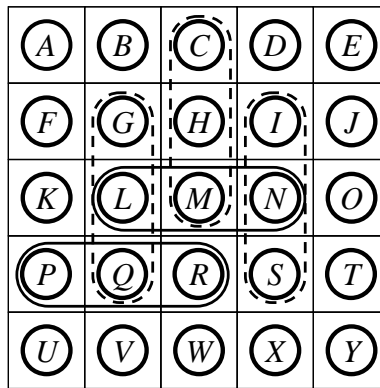


Fig. 4. Horizontal and vertical neighborhood structures for $N_{GA}(i)$ with three players.

It should be noted that we do not use the vertical neighborhood structure for $N_{IPD}(i)$. This is because the use of both the horizontal and vertical neighborhood structures for $N_{IPD}(i)$ is not consistent with the meaning of the interaction neighborhood. For example, $N_{IPD}(M) = \{L, M, N\}$ means that Player M plays against Player N while $N_{IPD}(N) = \{I, N, S\}$ means that Player N does not play against Player M . Thus we only use the horizontal neighborhood structure for $N_{IPD}(i)$ when $N_{IPD}(i)$ includes only three players. For example, $N_{IPD}(M) = \{L, M, N\}$ and $N_{IPD}(N) = \{M, N, O\}$ in Fig. 4.

Our computer simulations were performed with various specifications of a probability for a mistake (i.e., noise [2]), in which a player chooses an action different from the one suggested by its strategy. The idea of a noisy IPD model dates back to Lindgren [2]. We used the noisy IPD model with the standard pairing scheme in order to demonstrate the effect of using the two neighborhood structures on the evolution of cooperative behavior. As we show later in this section, cooperative behavior was

evolved independent of the use of the two neighborhood structures in the case of the mistake probability being zero.

We used the following parameter values:

Mistake probability: 0, 0.001, 0.01, 0.1,

Crossover probability: 1.0,

Mutation probability: $1/(5 \times 961)$,

Termination of the IPD game: 100 rounds,

Termination of the evolution: 1000 generations.

Under the mutation probability of $1/(5 \times 961)$, a single strategy is mutated at each generation on average since we have 961 players whose strategies are represented by binary strings of length five. The sensitivity of simulation results to the specifications of the crossover and mutation probabilities is examined later in this paper.

The average payoff over 100 independent runs for each combination of $N_{IPD}(i)$ and $N_{GA}(i)$ is shown in Fig. 5 for the case of the mistake probability 0.1. The average payoff in Fig. 5 was calculated over 1000 generations in each of 100 independent runs. An initial strategy for each player was generated by randomly generating a binary string of length five (i.e., by randomly specifying each bit value of the binary string as 0 and 1 with the same probability). We also show the percentage of the occurrence of mutual cooperation (C, C) in Fig. 6. That is, Fig. 6 shows the percentage of the rounds with the mutual cooperation (C, C) among all rounds of the game. These figures show that the size of the neighborhood for interaction (i.e., the number of players in $N_{IPD}(i)$) has a significant effect on the evolution of cooperative behavior. Higher payoffs and more frequent mutual cooperation were obtained from a smaller neighborhood for interaction. The size of the neighborhood for mating (i.e., the number of players in $N_{GA}(i)$) also has an effect on the evolution of cooperative behavior while its effect is much smaller than that of the size of the interaction neighborhood. In Fig. 7, we show the average payoff at the 1000-th generation. We can see that the average results at the 1000-th generation in Fig. 7 are similar to those over 1000 generations in Fig. 5. Good results were not obtained even at the final generation in Fig. 7 when the interaction neighborhood $N_{IPD}(i)$ was large.

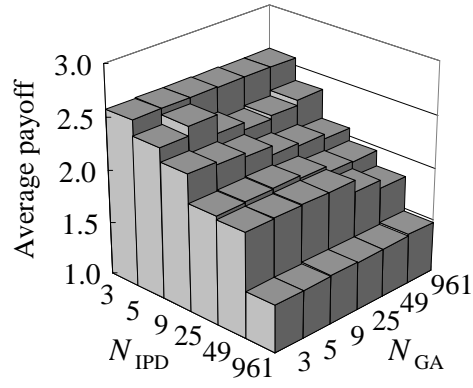


Fig. 5. Average payoff over 1000 generations in the case of the mistake probability 0.1.

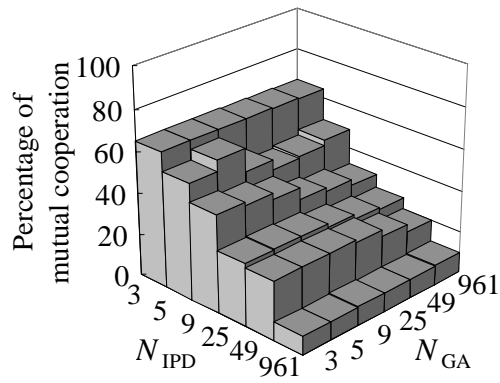


Fig. 6. Percentage of mutual cooperation over 1000 generations in the case of the mistake probability 0.1.

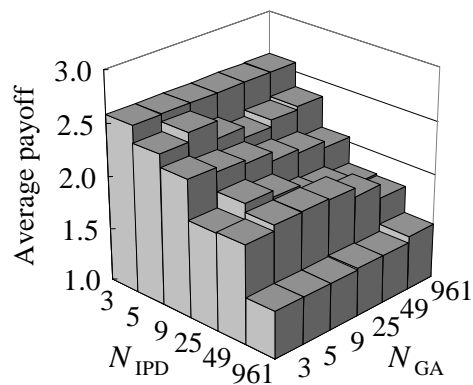


Fig. 7. Average payoff at the 1000-th generation in the case of the mistake probability 0.1.

Simulation results with other specifications of the mistake probability are shown in Fig. 8 and Fig. 9. These figures show that better results were obtained from smaller mistake probabilities. When we specified the mistake probability as zero, good results were obtained independent of the specifications of the two neighborhood structures. In this case, the average payoff and the percentage of mutual cooperation were larger than 2.9 and 95%, respectively, for all the 36 combinations of $N_{IPD}(i)$ and $N_{GA}(i)$.

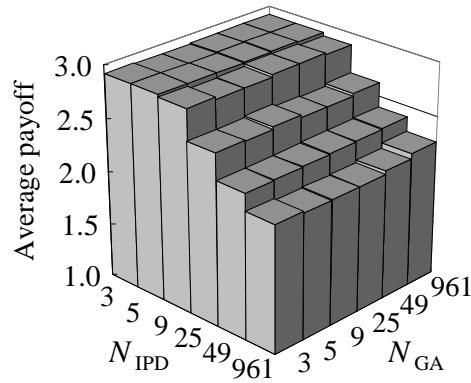


Fig. 8. Average payoff over 1000 generations in the case of the mistake probability 0.01.

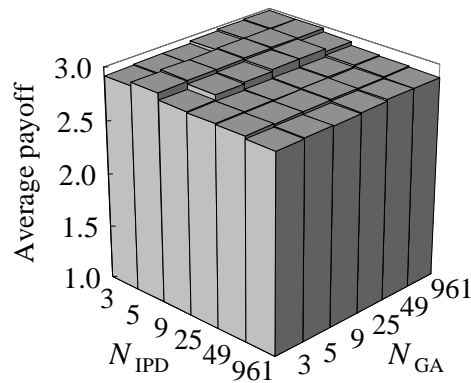


Fig. 9. Average payoff over 1000 generations in the case of the mistake probability 0.001.

In order to examine the effect of the crossover and mutation operations on the evolution of cooperative behavior, we performed the same computer simulation as Fig. 5 by specifying the crossover or mutation probability as zero. Simulation results are shown in Fig. 10 (no mutation) and Fig. 11 (no crossover). When the mutation probability was zero, simulation results depend strongly on

randomly generated initial strings and a sequence of random numbers used in each trial. Thus there are a number of ups and downs in Fig. 10 with no mutation. On the other hand, Fig. 11 with only mutation (i.e., no crossover) is very similar to Fig. 5 with both crossover and mutation. In both Fig. 10 with only crossover and Fig. 11 with only mutation, best results were obtained in the case of the smallest interaction neighborhood $N_{IPD}(i)$ with three players (i.e., $|N_{IPD}(i)| = 3$).

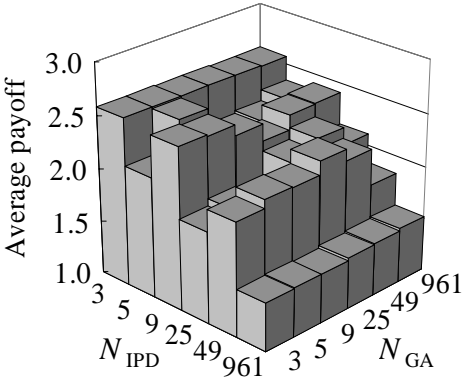


Fig. 10. Average payoff over 1000 generations in the case of the mutation probability being zero. The other parameter specifications are the same as Fig. 5.

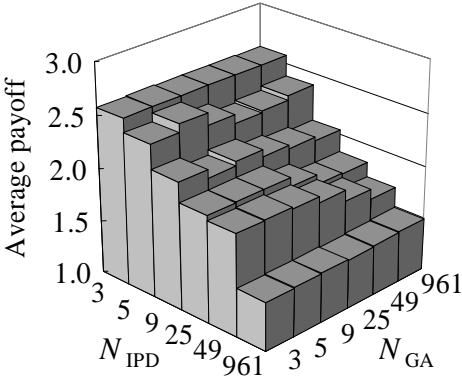


Fig. 11. Average payoff over 1000 generations in the case of the crossover probability being zero. The other parameter specifications are the same as Fig. 5.

We also examined the effect of random sampling of five opponents from $N_{IPD}(i)$ on our simulation results. For this purpose, we performed the same computer simulation as Fig. 5 without this

sampling. That is, each player played against all players in $N_{\text{IPD}}(i)$. This computer simulation was very time-consuming. For example, when $N_{\text{IPD}}(i)$ includes 961 players, it requires about 200 times more CPU time than the case of random sampling of five opponents. The average payoff over ten independent runs for each combination of $N_{\text{IPD}}(i)$ and $N_{\text{GA}}(i)$ is shown in Fig. 12. We can see that similar results were obtained in the two settings of game playing: Fig. 5 with random sampling of five opponents from $N_{\text{IPD}}(i)$ and Fig. 12 with game-playing against all opponents in $N_{\text{IPD}}(i)$.

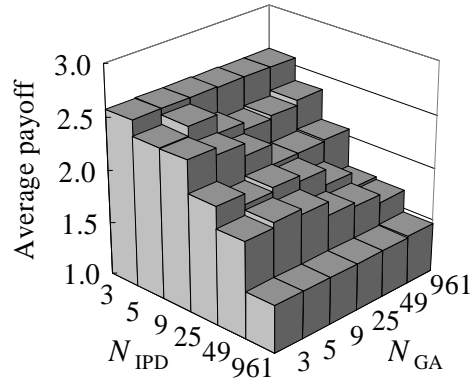


Fig. 12. Average payoff over 1000 generations with game-playing against all opponents in $N_{\text{IPD}}(i)$.

The other parameter specifications are the same as Fig. 5.

IV. IPD Game with Random Pairing

In this section, we change the pairing scheme in the spatial IPD game to further examine the effect of the two neighborhood structures on the evolution of cooperative behavior. In the previous section and almost all studies on the IPD game in the literature, each player plays against the same opponent for a prespecified number of rounds (e.g., 100 rounds in our computer simulations in the previous section). We change this pairing scheme in the following manner: Every player chooses its opponent randomly from $N_{\text{IPD}}(i)$ at every round of the game. This pairing scheme requires a new implementation because the game is not iterated between the fixed pair of players for a prespecified number of rounds. This random pairing scheme may correspond to a simple model of territorial animals with partially overlapping territories. In this model, every territorial animal is supposed to walk randomly about in its territory. When an animal comes across its neighbor, some interaction among them may happen just once. The memory about the interaction with a neighbor may influence an animal's future action against another neighbor. We have a number of similar situations in our

everyday life. For example, a tourist usually does not have an iterated interaction with a souvenir store owner at a foreign sightseeing spot. In this case, the memory about the interaction with the owner may have an effect on future actions of the tourist at other souvenir stores. While a buyer in online auction sites usually does not buy from the same seller many times, memories about past transactions with some sellers will influence future actions of the buyer in transactions with other sellers. Random pairing seems to be more suitable to these situations than the iterated interaction with the same opponent.

In the spatial IPD game with random pairing, the choice of an action by a player depends on the previous action of its opponent, which is in general different from the opponent in the current round. For example, the defection of an opponent in the previous round may cause the defection of the player against a different opponent in the current round. We implement the spatial IPD game with the random pairing scheme in the 31×31 grid-world as follows:

- Step 0:** Specify t as $t = 1$ where t indexes the number of rounds of the IPD game. Let T be the maximum number of rounds of the IPD game, which is used as the termination condition of the execution of the IPD game.
- Step 1:** Specify i as $i = 1$ where i is a player index.
- Step 2:** Randomly select one player (say Player j) from $N_{\text{IPD}}(i)$.
- Step 3:** Player i plays a single round of the IPD game against Player j based on their strategies.
- Step 4:** Update the memories of Player i and Player j according to the result of the game in Step 3.
- Step 5:** If $i < 961$ (i.e., if some players have not been selected as Player i yet), let $i := i + 1$ and return to Step 2.
- Step 6:** If $t < T$, let $t := t + 1$ and return to Step 1. Otherwise stop the execution of the IPD game.

By the above procedures, the fitness values of all players are calculated simultaneously. The next population of strategies is generated by the genetic operations with the neighborhood structure $N_{\text{GA}}(i)$ using the calculated fitness values.

The same parameter specifications were used as in the previous computer simulations in Section III. Since opponents are selected randomly at every round, the evolution of reciprocal strategies is very difficult to achieve. Actually the average payoff was 1.01 even when we specified the mistake probability as zero in the non-spatial IPD game with random pairing. The average payoff of 1.01 means that mutual defection was played in almost all rounds.

The average payoff over 1000 generations of 100 independent runs is shown in Fig. 13 for each combination of the two neighborhood structures in the spatial IPD game with the random pairing scheme. The mistake probability was specified as zero in Fig. 13. High average payoff was obtained only when we used $N_{\text{IPD}}(i)$ with three players and $N_{\text{GA}}(i)$ with three, five, or nine players. We also

show percentage of mutual cooperation in Fig. 14, which shows that the use of the smallest interaction neighborhood and a small mating neighborhood facilitated the evolution of cooperative behavior. Cooperative behavior was not evolved when the interaction neighborhood $N_{IPD}(i)$ included nine or more players. In Fig. 15, we show average results at the 1000-th generation. Even at the 1000-th generation, the average payoff was close to 1.00 when the interaction neighborhood $N_{IPD}(i)$ included nine or more players. We can see that the average results over 1000 generations in Fig. 13 are similar to those at the 1000-th generation in Fig. 15.

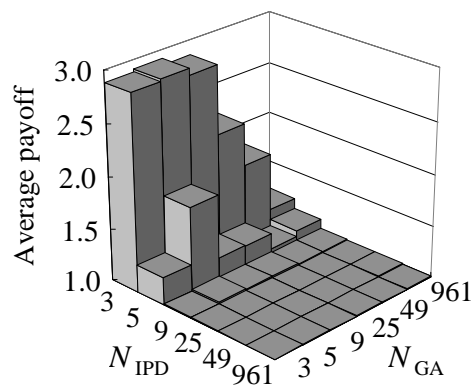


Fig. 13. Average payoff over 1000 generations with random pairing in the case of the mistake probability zero.

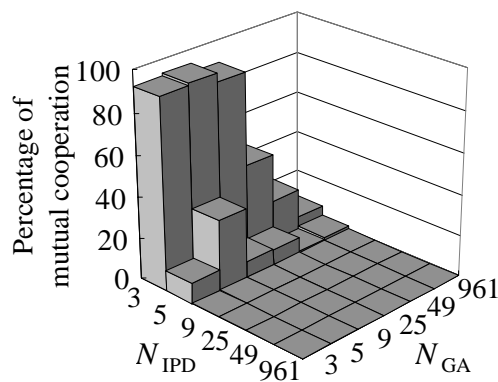


Fig. 14. Percentage of mutual cooperation over 1000 generations with random pairing in the case of the mistake probability zero.

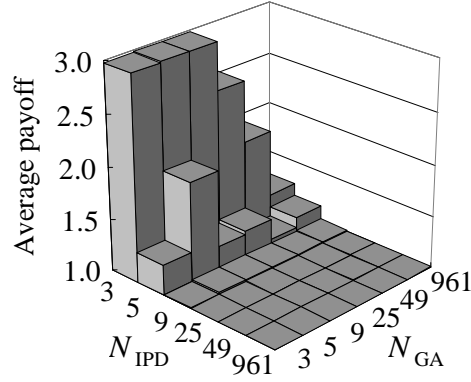


Fig. 15. Average payoff at the 1000-th generation with random pairing in the case of the mistake probability zero.

Similar results were obtained in the case of other specifications of the mistake probability. For example, the average payoff over 1000 generations of 100 independent runs is shown in Fig. 16 for the case of the mistake probability 0.01. As in Fig. 13 with the mistake probability zero, high average payoff was obtained when we used the smallest interaction neighborhood for $N_{IPD}(i)$ and a small mating neighborhood for $N_{GA}(i)$.

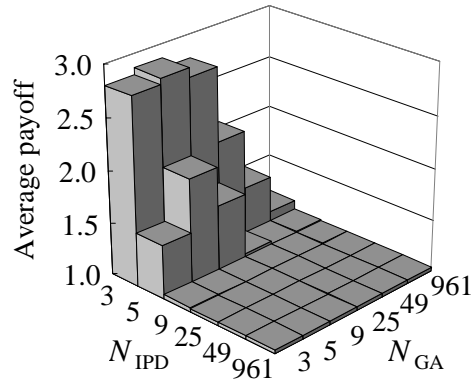


Fig. 16. Average payoff over 1000 generations with random pairing in the case of the mistake probability 0.01.

As we have already explained in Section III using Fig. 4, we used only the horizontal neighborhood structure as the smallest interaction neighborhood $N_{IPD}(i)$ while both the horizontal and vertical neighborhood structures were used as the smallest mating neighborhood $N_{GA}(i)$ in the case of

$|N_{\text{IPD}}(i)| = 3$ and $|N_{\text{GA}}(i)| = 3$. For comparison, we examined the use of both the horizontal and vertical neighborhood structures as the smallest interaction neighborhood $N_{\text{IPD}}(i)$ in the computer simulation in Fig. 13. In this computer simulation, we randomly assigned the horizontal or vertical neighborhood structure to each player in the 31×31 grid-world as $N_{\text{IPD}}(i)$. Table 3 compares average payoff obtained from the two specifications of $N_{\text{IPD}}(i)$. In Table 3, we observe the decrease in average payoff by the use of both the horizontal and vertical neighborhood structures as $N_{\text{IPD}}(i)$. This decrease can be explained as follows. When we used both the horizontal and vertical neighborhood structures as $N_{\text{IPD}}(i)$ in the case of $|N_{\text{IPD}}(i)| = 3$, each player played against more than three opponents as we have already explained in Section III using Fig. 4. This increase in the number of opponents decreased the probability to play against the same opponent in the next round of the IPD game under the random pairing scheme. The decrease in the probability to play against the same opponent also explains the decrease in average payoff with the increase in the size of the interaction neighborhood $N_{\text{IPD}}(i)$ in Fig. 13.

Table 3. Average payoff obtained from the two specifications of $N_{\text{IPD}}(i)$ in the case of $|N_{\text{IPD}}(i)| = 3$ (i.e., the use of only the horizontal neighborhood structure and the use of both the horizontal and vertical neighborhood structures). The other specifications are the same as Fig. 13.

$N_{\text{IPD}}(i)$	Size of $N_{\text{GA}}(i)$						Average
	3	5	9	25	49	961	
Horizontal	2.89	2.91	2.85	2.19	1.78	1.21	2.31
Both	2.44	2.50	2.29	1.66	1.53	1.22	1.94

In order to examine the effect of the size of the two-dimensional grid-world on our simulation results, we performed the same computer simulation as Fig. 13 using a 100×100 grid-world with 10000 players. Simulation results of 100 independent runs for each combination of $N_{\text{IPD}}(i)$ and $N_{\text{GA}}(i)$ are summarized in Fig. 17. While average payoff was increased by the use of the larger grid-world for some combinations of $N_{\text{IPD}}(i)$ and $N_{\text{GA}}(i)$ in Fig. 17 from Fig. 13, the same conclusion can be obtained from Fig. 13 and Fig. 17. That is, high average payoff was obtained when we used the smallest interaction neighborhood $N_{\text{IPD}}(i)$ and a small mating neighborhood $N_{\text{GA}}(i)$ with three, five, or nine players.

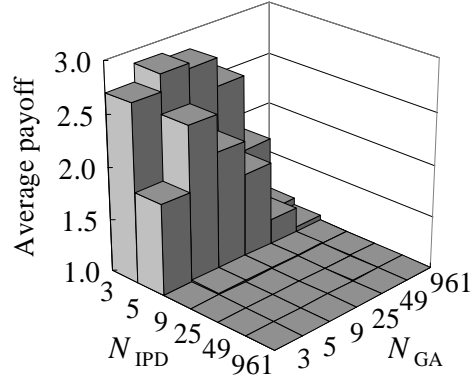


Fig. 17. Average payoff over 1000 generations with random pairing in the 100×100 grid-world. The same parameter specifications were used as Fig. 13.

In order to demonstrate how cooperative behavior was evolved with random pairing in the computer simulation of Fig. 13 in the 31×31 grid-world with the mistake probability zero, we show percentages of some strategies over 100 independent runs for two combinations of $N_{IPD}(i)$ and $N_{GA}(i)$ in Fig. 18 and Fig. 19. Strategies characterized by the generic form “1***1” had high percentages in Fig. 18 (with $|N_{IPD}(i)| = 3$ and $|N_{GA}(i)| = 5$) where high average payoff was obtained. Strategies of this form (e.g., TFT “10011” and Pavlov “11001” [28]) start with cooperation (C) and cooperate when the result of the previous round was mutual cooperation (C, C). Thus 100% mutual cooperation can be achieved under the mistake probability zero when all players have strategies of this type, even in the case of random pairing. In Fig. 20, we show how the average percentage of mutual cooperation over 100 independent runs in Fig. 18 increased during 1000 generations.

On the other hand, strategies of the form “****0” were included in Fig. 19 (with $|N_{IPD}(i)| = 5$ and $|N_{GA}(i)| = 5$) where average payoff was not high. Strategies of this form defect when the result of the previous round was mutual cooperation (C, C). Thus the existence of strategies of this type prevents the consecutive occurrence of mutual cooperation. As a result, the average payoff and percentage of mutual cooperation were not high in the trials shown in Fig. 19, where percentages of those strategies were high. Fig. 21 shows the average percentage of mutual cooperation over 100 independent runs in Fig. 19.

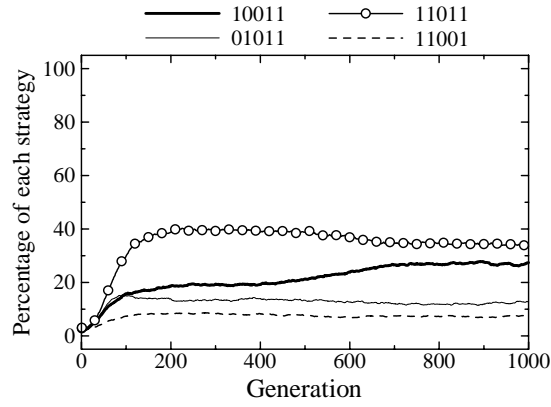


Fig. 18. Percentages of some strategies under the random pairing scheme, the mistake probability zero, $N_{IPD}(i)$ with three players, and $N_{GA}(i)$ with five players. Average payoff was 2.91.

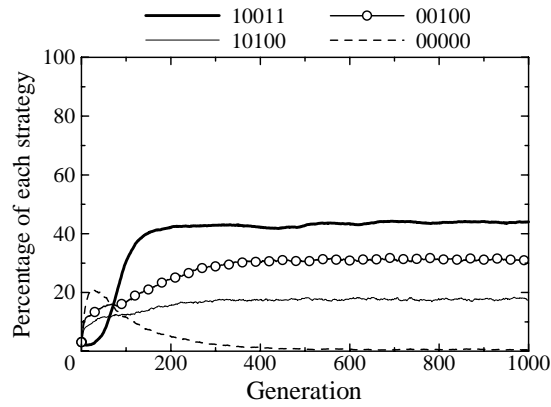


Fig. 19. Percentages of some strategies under the random pairing scheme, the mistake probability zero, $N_{IPD}(i)$ with five players, and $N_{GA}(i)$ with five players. Average payoff was 1.82.

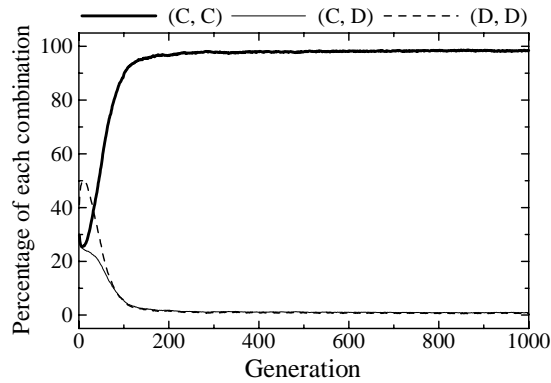


Fig. 20. Percentages of mutual cooperation (C, C), and mutual defection (D, D) over the same 100 runs in Fig. 18. We also show the percentage of (C, D) which includes the case of (D, C).

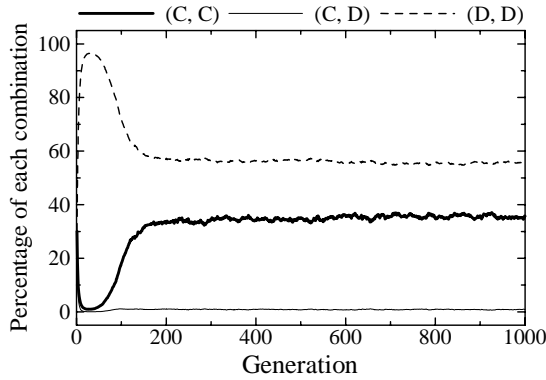


Fig. 21. Percentages of mutual cooperation (C, C), and mutual defection (D, D) over the same 100 runs in Fig. 19. We also show the percentage of (C, D) which includes the case of (D, C).

Percentages of some strategies were also examined in the case of the mistake probability 0.01 (i.e., in the computer simulation of Fig. 16) over 100 independent runs for the same two combinations of $N_{IPD}(i)$ and $N_{GA}(i)$ as in Fig. 18 and Fig. 19. Results are shown in Fig. 22 and Fig. 23. For the combination of $|N_{IPD}(i)| = 3$ and $|N_{GA}(i)| = 5$ that showed high average payoff (i.e., Fig. 22), strategies characterized by the generic form “11**1” had high percentages. Strategies of this form (e.g., Pavlov “11001” [28]) start with cooperation (C) and cooperate when the result of the previous round was mutual cooperation (C, C) and mutual defection (D, D). That is, those strategies have the ability to recover from mutual defection. This ability seems to be important under a noisy situation with the existence of mistakes. The TFT strategy “10011”, which does not have this ability, decreased its percentage during the evolution of cooperative behavior under the existence of mistakes in Fig. 22 while it maintained high percentage in the case of the mistake probability zero in Fig. 18. High percentage of mutual cooperation was achieved in Fig. 22. Fig. 24 shows how the average percentage of mutual cooperation increased during 1000 generations of 100 independent runs in Fig. 22.

On the other hand, we can observe that the TFT strategy “10011” increased its percentage to almost 100% during 1000 generations in Fig. 23. This does not always mean the evolution of cooperative behavior under random pairing in game-playing with the existence of mistakes. Fig. 25 shows the average percentage of mutual cooperation over 100 independent runs in Fig. 23. While almost all players used the TFT strategy at the 1000-th generation in Fig. 23, the corresponding percentage of mutual cooperation was about 40% in Fig. 25. As shown in Fig. 22, a higher average payoff was obtained from strategies of the form “11**1” rather than the TFT strategy “10011”.

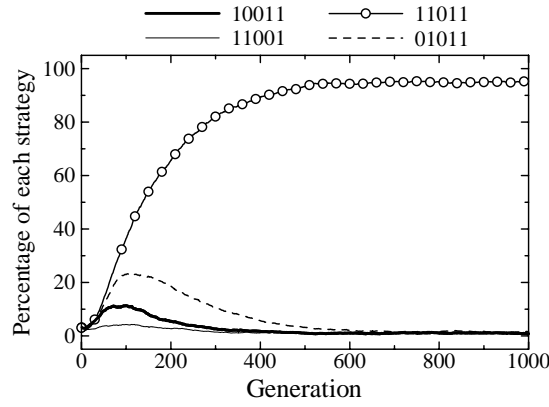


Fig. 22. Percentages of some strategies under the random pairing scheme, the mistake probability 0.01, $N_{IPD}(i)$ with three players, and $N_{GA}(i)$ with five players. Average payoff was 2.87.

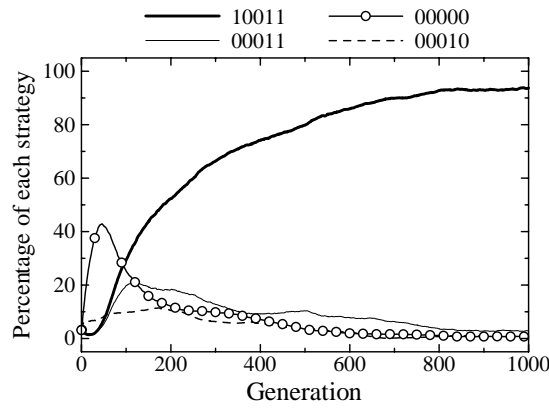


Fig. 23. Percentages of some strategies under the random pairing scheme, the mistake probability 0.01, $N_{IPD}(i)$ with five players, and $N_{GA}(i)$ with five players. Average payoff was 2.03.

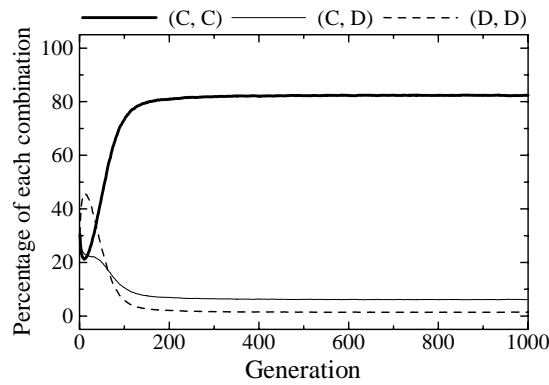


Fig. 24. Percentages of mutual cooperation (C, C), and mutual defection (D, D) over the same 100 runs in Fig. 22. We also show the percentage of (C, D) which includes the case of (D, C).

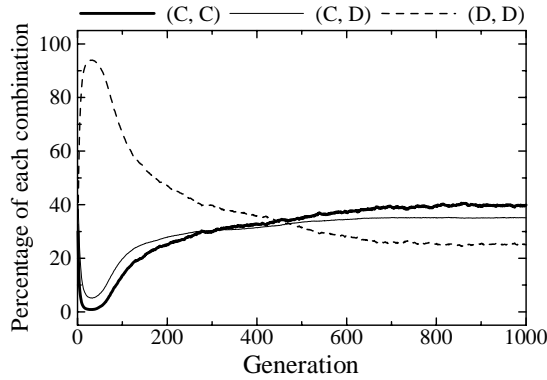


Fig. 25. Percentages of mutual cooperation (C, C), and mutual defection (D, D) over the same 100 runs in Fig. 23. We also show the percentage of (C, D) which includes the case of (D, C).

We further examined the 36 combinations of the two neighborhood structures for various specifications of the mistake probability (i.e., 0, 0.001, 0.005, 0.01, 0.05, 0.1). Simulation results for the mistake probabilities 0 and 0.01 were shown in Fig. 13 and Fig. 16, respectively. In our computer simulations with various specifications of the mistake probability, the highest average payoff was always obtained from $N_{IPD}(i)$ with three players and $N_{GA}(i)$ with five players. Table 4 summarizes the relation between the mistake probability and the best average payoff among the 36 combinations of the two neighborhood structures. As in the case of the standard pairing scheme with a long interaction sequence against the same opponent, Table 4 shows that high mistake probabilities decreased the average payoff. That is, the evolution of cooperative behavior was disturbed by high mistake probabilities in the case of the random pairing scheme as well as the standard one.

Table 4. Relation between the mistake probability and the best average payoff among the 36 combinations of the two neighborhood structures under the random pairing scheme.

Probability	0	0.001	0.005	0.01	0.05	0.1
Payoff	2.91	2.92	2.89	2.87	2.71	2.56

We also examined the effect of the crossover and mutation probabilities on the evolution of cooperative behavior under random pairing. The mistake probability was specified as zero and the size of the interaction neighborhood $N_{IPD}(i)$ as three (i.e., $|N_{IPD}(i)| = 3$). We examined four specifications of the crossover probability: $P_C = 0, 0.1, 0.5, 1.0$, and also four specifications of the mutation probability: $P_M = 0, 0.1/(5 \times 961), 1/(5 \times 961), 10/(5 \times 961)$. We show percentages of

mutual cooperation for various specifications of the crossover and mutation probabilities in Table 5 and Table 6 where high percentages of mutual cooperation (over 70%) are highlighted by boldface. Table 5 shows the effect of the crossover probability on the percentage of mutual cooperation. High percentages of mutual cooperation were obtained in Table 5 even in the case of the crossover probability being zero. On the other hand, Table 6 shows the effect of the mutation probability on the percentage of mutual cooperation. High percentages of mutual cooperation were obtained in Table 6 in a wide range of mutation probabilities while a too large mutation probability decreased the percentage of mutual cooperation.

Table 5. Percentage of mutual cooperation for each specification of the crossover probability ($|N_{IPD}(i)| = 3$ and $P_M = 1/(5 \times 961)$).

Crossover probability	Size of $N_{GA}(i)$						Average
	3	5	9	25	49	961	
0.0	93.4	95.6	93.2	63.4	42.0	37.0	70.8
0.1	92.7	95.1	92.7	57.7	26.7	11.1	62.7
0.5	92.4	94.1	90.6	44.8	21.1	5.2	58.0
1.0	92.3	93.6	90.2	40.5	19.6	3.3	56.6

Table 6. Percentage of mutual cooperation for each specification of the mutation probability ($|N_{IPD}(i)| = 3$, $P_C = 1$ and $P_M = 1/(5 \times 961)$).

Mutation probability	Size of $N_{GA}(i)$						Average
	3	5	9	25	49	961	
0.0	94.0	94.5	90.3	36.2	16.3	2.4	55.6
$0.1 \times P_M$	93.8	94.3	90.7	37.9	18.6	3.8	56.5
P_M	92.3	93.6	90.2	40.5	19.6	3.3	56.6
$10 \times P_M$	71.9	81.2	70.7	37.7	16.7	3.9	47.0

We also performed computer simulations using other specifications of the interaction neighborhood $N_{IPD}(i)$. High percentages of mutual cooperation were not obtained when $N_{IPD}(i)$ included five neighbors. More specifically, the percentage of mutual cooperation was always less than 50% in the case of $|N_{IPD}(i)| = 5$. When $N_{IPD}(i)$ included nine or more neighbors, the percentage of mutual cooperation was always almost zero. Since these results are similar to Fig. 13, they are omitted in Table 5 and Table 6. These tables show that the combination of the smallest interaction neighborhood $N_{IPD}(i)$ and a small mating neighborhood $N_{GA}(i)$ facilitated the evolution of cooperative behaviors

in all the examined specifications of the crossover and mutation probabilities.

Finally we examined other specifications of the number of rounds (i.e., iterations) of the game in each generation for each player. In each trial of the computer simulations in this section, we specified the number of rounds in each generation as 100 for each player (i.e., $T = 100$ in the random pairing scheme). We performed computer simulations using other specifications of the number of rounds (i.e., $T = 10, 20, 50, 1000$). Average results over 100 independent runs are shown in Table 7. We can see that similar results were obtained in Table 7 from the five specifications of the number of rounds. We can also see that the increase in the number of rounds slightly increased the percentage of mutual cooperation.

Table 7. Percentage of mutual cooperation for each specification of the number of rounds ($|N_{IPD}(i)| = 3$, $P_C = 1$ and $P_M = 1/(5 \times 961)$).

Number of rounds (T)	Size of $N_{GA}(i)$						Average
	3	5	9	25	49	961	
10	93.7	92.8	90.3	36.2	12.6	5.4	55.2
20	93.1	92.0	88.7	40.1	14.8	5.8	55.7
50	92.5	92.8	89.3	41.5	15.9	4.7	56.1
100	92.3	93.6	90.2	40.5	19.6	3.3	56.6
1000	91.1	94.3	89.8	53.8	20.8	9.4	59.9

V. Conclusions

In this paper, we first formulated a spatial IPD game using the concept of structured demes. The main characteristic of our spatial IPD game is the use of two neighborhood structures: One is for the interaction among players through the IPD game and the other is for mating strategies. Next we demonstrated through computer simulations that the use of a small interaction neighborhood facilitated the evolution of cooperative behavior even in the situation with a relatively high mistake probability (e.g., 0.1). In the case of the mistake probability 0.1, the percentage of mutual cooperation was high (i.e., about 65%) with the smallest interaction neighborhood including only three players while it was low (i.e., about 10%) with no neighborhood structures. On the other hand, when the mistake probability was zero, the evolution of cooperative behavior was always evolved for all combinations of the two neighborhood structures, including the case of no neighborhood structures.

Then we introduced a random pairing scheme with the two neighborhood structures, in which a player chose a different opponent in each round of the game randomly from its interaction neighborhood. Simulation results demonstrated that cooperative behavior was evolved when we used

the smallest interaction neighborhood with three players and a small mating neighborhood with three, five, or nine players. When the interaction neighborhood included nine or more players, the percentage of mutual cooperation was almost zero and the average payoff was about one, which means that mutual defection was always played. Further analysis is required to explain these results.

In this paper, a player's strategy was represented by a binary string. An interesting extension is to use a stochastic strategy represented by a string of real numbers between 0 and 1 (e.g., Brauchli et al. [16]). Each real number in the string denotes the probability of cooperation. The evolution of stochastic strategies in the spatial IPD game with random pairing is left for a future research topic.

Another future research topic is the evolution of cooperative behavior under the random pairing scheme in a large interaction neighborhood. In this situation, cooperative behavior was not evolved in our computer simulations. A promising approach would be to tag players with certain types. Each player may have a strategy for responding to players of each type.

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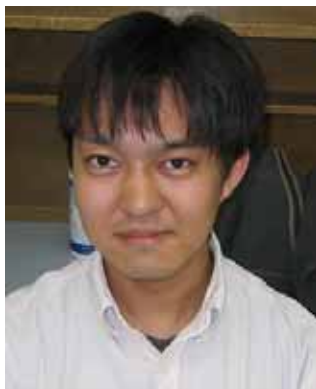
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